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Band gaps and transmission spectra in generalized Fibonacci $\sigma (p, q)$ one-dimensional magnonic quasicrystals

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Abstract

We employ a microscopic theory to investigate spin wave (magnon) propagation through their dispersion and transmission spectra in magnonic crystals arranged to display deterministic disorder. In this work the quasiperiodic arrangement investigated is the well-known generalized Fibonacci sequence, which is characterized by the $\sigma (p, q)$ parameter, where $p$ and $q$ are non-zero integers. In order to determine the bulk modes and transmission spectra of the spin waves, the calculations are carried out for the exchange dominated regime within the framework of the Heisenberg model and taking into account the random phase approximation. We have considered magnetic materials that have a ferromagnetic order, and the transfer-matrix treatment is applied to simplify the algebra. The results reveal that spin wave spectra display a rich and interesting magnonic pass- and stop-bands structures, including an almost symmetric band gap distribution around of a mid-gap frequency, which depends on the Fibonacci sequence type.

1. Introduction

In recent years, many interesting properties, new alternatives for traditional materials and effects have been observed in magnetic materials, for example topological insulators [1, 2], spintronics [3–5] and the spin Seebeck effect [6, 7]. In this context, a new research field has attracted the attention of many theoretical and experimental research groups. This field comprises artificial structures made by magnetic materials in which the magnetic properties change periodically in space and, thus, the spin wave (SW) propagation can be modulated and controlled. These have been called magnonic crystals (MCs) [8], in analogy to their electronic and electromagnetic counterparts, i.e. electronic (ECs) and photonic crystals (PCs), respectively, and because magnons are the quasiparticles related to SWs. More commonly, MCs can also be defined as being the magnetic analogues of PCs, which are dielectric microstructures with modulated periodicity which controls the electromagnetic wave (more precisely, photon) propagation and which exhibit the unique property of photonic band gaps (PBGs), i.e. regions of the dispersion spectrum where photon propagation is prohibited [9]. Similar to PCs, magnonic structures can also display frequency regions in which magnons with a given wavevector do not propagate in this material. Such regions are called magnonic band gaps (MBGs) [10].

Many efforts has been made to investigate both theoretically [11–13] and experimentally [14, 15] the magnetic structure in the magnetostatic regime (MSR), i.e. a regime in which the interaction among the magnetic momenta is predominantly dipolar and propagation of SWs is well described by using a macroscopic theory—frequently by employing the Landau–Lifshitz–Gilbert (LLG) magnetization equation, for example. On the other hand, magnetic
heterostructures in the exchange regime (ER), in which microscopic interaction makes the main contribution to the internal energy of a magnetic system, has also been the subject of considerable theoretical interest in the context of MCs [16–18]. Recently, Costa et al. [19] considered one-dimensional magnonic crystals in the ER arranged in the \( z \)-direction and it verified the existence of partial magnonic band gaps (PMBGs) [20], i.e., the magnonic band gap appears only in some wavevector directions, in this case \( M \rightarrow R \) direction (which corresponds to the \( z \)-direction in the real space). For magnons in the exchange regime it is necessary to take into account that the exchange terms of magnetic materials display the same role as the permeability function in magnetostatic MCs. The main interest in studying MCs in the ER is the fact that magnetic excitations can propagate with a frequency range of some tens of terahertz (THz) [19], while the frequency range of magnetostatic SWs can reach hundreds of gigahertz (GHz) [8]. Besides this, one-dimensional MCs fabricated on the atomic scale are completely realizable experimentally thanks to modern growth techniques like molecular beam epitaxy (MBE) [21]; they are therefore the simplest system in which MBGs can be experimentally studied.

Another research field of mathematical [22], physical [23] and biological [24] interest investigated in this paper is substitutional sequences, which are one class of the many systems that belong to the so-called quasicrystals [25]. The subject of quasicrystals gained prominence in 1984 with the discovery of crystalline phases that had previously been forbidden by crystallographic rules, i.e., quasicrystals do not belong to the set of Bravais lattices. In this case, Shechtman et al. [26] presented interesting and surprising electron diffraction patterns of metallic aluminum and magnesium (Al–Mn) alloys. They melted Al and Mn in the proportion of six-to-one and the alloy was quickly cooled and mixed using a melt spinning process. Under electron microscopy, the solidified alloy displayed a new crystalline structure with five-fold symmetry, thus such systems do not have translation symmetry. This structure also revealed a long-range atomic order. It was also observed that this order is not completely amorphous or periodic and, because of this, it was mainly considered as a suitable theoretical model to describe the conceptual transition from a perfect periodic structure to a random one. Hence, frequently, one says that quasicrystals are a phase between the crystalline and aperiodic regimes. After this, other forbidden symmetries were also found in other materials, such as icosahedral symmetry. Such diffraction spectra were theoretically explained when Levine and Steinhardt [27] investigated these symmetries through Penrose tiling, which is a geometric tool for filling the space using regular polygons (2D) or polyhedra (3D). For such amazing discovery Daniel Shechtman was awarded with the Nobel Prize in Chemistry in 2011.\(^3\)

The non-periodic but deterministic quasiperiodic structures constitute a prominent research area. The most popular substitutional sequences are Fibonacci, Thue–Morse, Rudin–Shapiro and double-period [28] and also the fractal structures like Cantor sets or Koch fractals [29]. Quasiperiodic structures have a fascinating feature, which is that collective properties presented by them are not shared by their constituent parts. Beside of this, such properties also are distinct from periodic structures. Furthermore, the long-range correlations induced by the construction of these systems are expected to be reflected to some degree in their various spectra, defining a novel description of disorder [30]. Some theoretical treatments, like the transfer-matrix method (TMM), show that these spectra are fractals. MBGs were studied in quasiperiodic magnonic crystals or magnonic quasicrystals (MQCs) without [31] and with uniaxial anisotropy [32], and including non-linear effects [33]. MQCs are defined like MCs, in that the spatial distribution of the exchange terms is arranged in a quasiperiodic fashion, obeying mathematical rules, which in the present work are the generalized Fibonacci sequences (GFSs): golden (GM), silver (SM), bronze (BM), copper (CM) and nickel (NM) mean.

The GFSs can be obtained by an inflation rule or recursive sequence, forming a binary string that can be grown by juxtaposing two building blocks \( A \) and \( B \). The \( n \)th stage \( S_n \) of MQCs is generated by \( S_n = S_{n-1}^p S_{n-2}^q \) (\( n \geq 2 \)), with \( S_0 = B \) and \( S_1 = A \). The indices \( p \) and \( q \) are arbitrary positive integer numbers. Here, \( S_n^{p(q)} \) represents \( p(q) \) adjacent repetitions of the stack \( S_n \). Another way to obtain a GFS is through the recurrence relation \( A \rightarrow A^p B^q \) and \( B \rightarrow A \), where \( A^p(B^q) \) means a string of \( p(q) \) \( A(B)s \), as is shown in figure 1. We remember that for \( p = q = 1 \) we recover the ordinary Fibonacci sequence. The total number of blocks \( A \) and \( B \) in \( S_n \) is equal to the generalized Fibonacci number \( F_n \), and is given by the recurrence relation

\[
F_n = pF_{n-1} + qF_{n-2},
\]

with \( F_0 = F_1 = 1 \). In this sequence one can also define the characteristic irrational value \( \sigma(p,q) \), which is the ratio \( F_n \) to \( F_{n-1} \) when \( n \rightarrow \infty \), and it is given by the positive solution of the quadratic equation

\[
\sigma^2 - \sigma - q = 0,
\]

or explicitly by

\[
\sigma(p,q) = \lim_{n \to \infty} \frac{F_n}{F_{n-1}} = \frac{p + \sqrt{p^2 + 4q}}{2}.
\]

More details about GFSs can be found in [34, 35].

For each value of \( p \) and \( q \) there is a different characteristic value \( \sigma(p,q) \) which denotes the substitutional structure in question. When \( p = q = 1 \), we have \( \sigma(1,1) = \sigma_2 = (1 + \sqrt{5})/2 \), the well-known golden number. Similarly, \( \sigma(2,1) = \sigma_1 = 1 + \sqrt{2} \) is the silver number; \( \sigma(3,1) = \sigma_3 = (3 + \sqrt{13})/2 \) is the bronze number, \( \sigma(1,2) = \sigma_c = 2 \) is the copper number and \( \sigma(1,3) = \sigma_n = (1 + \sqrt{3})/2 \) is the nickel number (see figure 2). These values of \( \sigma \) are the most commonly known and they are called ‘metallic means’ [34]. Another way to classify the GFS sequences considered is based on the irrationality of \( \sigma^{-1}(p,q) \) (where

Figure 1. Schematic representation of the quasiperiodic MQC of the generalized Fibonacci type.

Figure 2. Plot of the ratio $F_n/F_{n-1}$ versus the generation number $n$ for the five GFSs studied here: GM (black square), SM (red circle), BM (green up triangle), CM (blue down triangle) and NM (dark green pentagon). We can observe that $F_n/F_{n-1}$ quickly converges to irrational parameters (represented by the dashed lines) $\sigma_g$ (black), $\sigma_s$ (red), $\sigma_b$ (green), $\sigma_c$ (blue) and $\sigma_n$ (dark green), given in table 1.

the minus signal means the negative root of equation (2)), i.e. if $|\sigma - (p, q)| < 1$, it is a Pisot–Vijayaraghavan (PV) irrational number [36], and the fluctuation of the physical properties of the substitution sequence is more accentuated. On the other hand, if $|\sigma - (p, q)| > 1$, it is not a PV-type number, and the fluctuation of its physical properties is less intense. In our case, only the CM and NM sequences are of non-PV-type and therefore we expect a more pronounced fluctuation of its physical properties. Actually, the GM, SM and BM sequences are quasiperiodic, while CM and NM cases represent only substitutional sequences! In table 1 we briefly present the sequences investigated together with their main characteristics. In the present paper we shall consider the generalized Fibonacci QCs defined by $p \geq 1$ and $q \geq 1$.

In this work we generalize previous results concerning MBGs, investigating the correspondence between magnon dispersion relation and transmission spectra in generalized Fibonacci structures considering GM, SM, BM, CM and NM characteristic irrational parameters.

This paper is organized as follows. In section 2 we present the theoretical modeling employed here, which is based on the transfer-matrix approach. The expressions for the SW dispersion relation (section 2.1) and transmission (section 2.2) are then determined, and they follow the pattern already shown in previous works (for a review, see [38, 39]). In section 3 we determine the general transfer-matrices for dispersion and transmission cases of SWs for any generation of generalized Fibonacci quasicrystals defined by $p \geq 1$ and $q \geq 1$. The numerical results are presented in section 4. Finally, the conclusions are given in section 5.

2. General theory

We use this section to present the theoretical methods employed to determine the relation dispersion and transmission spectra of SWs in one-dimensional MCs and MQCs. Basically, we use the Heisenberg Hamiltonian within the motion equation of the spin operator in the random phase approximation (RPA), together with a transfer-matrix approach to obtain the desired expressions. So, this section is divided in two subsections: in section 2.1 we present the equation for the SW dispersion relation, while in section 2.2 we show the equation that determines the SW transmission.
2.1. Transfer-matrix method for spin wave dispersion relation

We consider MCs in which $n_A$ layers of material A alternate with $n_B$ layers of material B. Initially, this alternation is in a periodic fashion (see figure 3(a)). Both materials are taken to be simple cubic spin-$S$ Heisenberg ferromagnetic materials, having bulk exchange constants $J_A$ and $J_B$ and lattice constant $a$. The exchange terms at the interfaces are $I$ (in the $A|B$ interface), $I_{A|B}$ (in the $A|A(B|B)$ interface), as is shown in figure 3(b).

The Heisenberg Hamiltonian for each component can be expressed as (we choose units in which $\hbar = 1$)

$$\mathcal{H} = -\frac{1}{2} \sum_{i \neq j} J_{ij} S_i \cdot S_j - g \mu_B \sum_i S_i \cdot H_0$$

(4)

where the first term is the exchange energy and the sum is over sites $i$ and nearest neighbors $j$ (the factor $1/2$ must be included because each pair of sites will be counted twice in the summation over $i$ and $j$), while the exchange term $J_{ij}$ is equal to $J_{A|B}$ in the bulk of material A (B), or $I$ ($I_{A|B}$) in the interface $A|B$ ($A|A(B|B)$, respectively). The second term represents the Zeeman effect, which describes the interaction between an external applied magnetic field $H_0$ (pointing in the $z$-direction) and a spin at the site $i$. The unit cell size is $D = na$, with $n = n_A + n_B$ (number of A-layers and B-layers in each cell). The $l$th unit cell is defined to run from $(l-1)na$ to $lna$.

The SW excitations were calculated from equation (4) by various different techniques. These systems are commonly solved using a method which involves transforming from spin operators to other operators such as boson creation and annihilation operators, the well-known Holstein–Primakoff (HP) transformation. However, in the present case it is more useful to work directly in terms of the spin operators, because in this framework many interesting effects, especially those that are temperature dependent, can be treated in a straightforward manner. For instance, at non-zero temperature the equilibrium configuration must exhibit the analog of surface reconstruction, i.e., the spins are not completely parallel, instead the spins are disordered. This implies that the mean spin $S$ in both materials is a function of its distance from the nearest interface (in the $A|B$ interface, for example). Although this effect is of interest it is not our concern here, and we circumvent it by restricting our attention to the low temperature regime, $T \ll T_C$ (where $T_C$ is the Curie temperature). In this regime, we can make use of the RPA [40], discussed in more detail below.

From elementary quantum mechanics, the equation of motion for the ladder spin operators $S_i^\pm = S_i^z \pm i S_i^x$, in the Heisenberg picture has the form

$$\frac{dS_i^\pm}{dt} = [S_i^\pm, \mathcal{H}]$$

(5)

By using the following commutation relations satisfied by the spin operators

$$[S_i^+, S_j^-] = 2S_i^z \delta_{ij} \quad \text{and} \quad [S_i^-, S_j^+] = \pm S_i^z \delta_{ij},$$

and equations (4) and (5) we obtain the motion equation for the spin operators $S_i^\pm$

$$\frac{dS_i^\pm}{dt} = g \mu_B H_0 S_i^z \pm \sum_j J_{ij} (S_j^+ S_i^- - S_i^+ S_j^-).$$

(6)

As there is no coupling term between $S_i^+$ and $S_i^-$, due to the absence of any anisotropic terms in the magnetic Heisenberg Hamiltonian, we can work arbitrarily here with either one of these operators [32]; in this case we choose the operator $S_i^+$. The product of spin operators in equation (6) may be simplified through the RPA, which basically consists of replacing the spin operator $S_i^z$ by its thermal average $\langle S_i^z \rangle$ (which, by symmetry, is the same at all sites), i.e.,

$$\left( S_i^+ S_i^- - S_i^- S_i^+ \right) \rightarrow \langle S_i^+ \rangle S_i^z - \langle S_i^- \rangle S_i^z.$$  

(7)

Substituting equation (7) in (6), the dispersion equation for SWs in a ferromagnetic medium (A or B) and in a magnetic superlattice can be found, within this linearization approximation, by solving

$$\frac{dS_i^z}{dt} = g \mu_B H_0 S_i^z + \sum_j J_{ij} \left( \langle S_j^+ \rangle S_i^- - \langle S_i^- \rangle S_j^+ \right).$$

(8)

Firstly, for bulk modes, $S_i^+$ is proportional to $e^{i(k \cdot r - \omega t)}$. Substituting this solution in equation (8), the dispersion relation of SWs in the bulk of a ferromagnet is (for more details, see [28])

$$\omega(k) = g \mu_B H_0 + 2 J_0 S_0 \left[ 3 - \gamma_1 - \cos(k \cdot a) \right].$$

(9)

where $\gamma_i = \cos(k_i a) + \cos(k_i a)$. In figure 4 we plot the solution of equation (9) for both materials A (red solid line) and B (blue dashed line) as a function of the reduced frequency $\Omega = \omega J_0 S_0$ versus the normalized dimensionless wavevector $k \cdot a / \pi$, where $k \cdot a$ is the wavevector in the $z$-direction times the lattice parameter $a$. In this figure,
Figure 3. Schematic representation (a) of the one-dimensional magnonic crystal investigated, which is a juxtaposition of the unit cell $[A|B]$, and (b) of the interface $A|B$ showing the disposition of the spins and the exchange interactions $J_{A|B}$, between spins in the bulk of medium $A(B)$, and $I$, between spins from different media.

we consider $k_za = k_za = 0$, the average spin $S_A = 1, S_B = 1.5, J_{AB} = J_A/J_B = 2, H_{0A} = g\mu_B H_0/J_A = 0$ (for negative $k_za$) and $H_{0A} = 1$ (for positive $k_za$).

Figure 4. Dispersion curve of spin waves in ferromagnetic materials (red solid line for material $A$ and blue dashed line for material $B$) as a function of the reduced frequency $\Omega = \omega J_A S_A$ versus the normalized dimensionless wavevector $k_za/\pi$. The parameters used are $k_za = k_za = 0$, the average spin $S_A = 1, S_B = 1.5, J_{AB} = 2$, $H_{0A} = 0$ (for negative $k_za$) and $H_{0A} = 1$ (for positive $k_za$).

We call attention to two important characteristics from equation (9) presented in figure 4. The first is that the only effect of an external applied magnetic field is the vertical shift of the dispersion curve. In other words, $H_0$ does not modify the shape of the dispersion curve. Because of this, from now on, we will not consider the Zeeman energy in the Hamiltonian. Second, for $H_{0A} = 0$, we can divide the spectrum in three regions: (i) in Region I ($0 \leq \Omega \leq 3$), $k_z$ assumes real values for both materials $A$ and $B$; (ii) in Region II ($3 < \Omega \leq 4$), $k_z$ in medium $A$ remains real, but in $B$ it now assumes complex values ($k_{zB} = \pi/a + i\delta$); and (iii) in Region III ($\Omega > 4$), the wavevector $k_z$ is complex for both materials $A$ and $B$. Albuquerque et al [41] has shown that most of allowed modes are in Region I, and the allowed bands are wider in this region too.

Now, in order to determine the dispersion relation in a periodic magnetic multilayer, we must solve equation (8) for spins localized at the interfaces. To do this, we propose a solution in which the SW amplitudes are given, within each material, by a linear combination of the positive- and negative-going solutions for the bulk medium, i.e. the solution for a spin at the site $i$ localized at the interface of material $A$ can be given by

$$S_i = \left\{ A_i e^{i|k_A|(r-r_{lA})} + A_i' e^{-i|k_A|(r-r_{lA})} \right\} \times e^{-i\omega t}. \tag{10}$$

For a spin situated on the interface of material $B$, one must change the amplitudes and indices from $A$ to $B$. Here, $r_{lA} = [(l+1)na + a] \hat{z}$ and $r_{lB} = [(l+1)na + (n+1)1a] \hat{z}$ are the positions of the left-hand layers of the corresponding component in cell $l$ in $A$ and $B$, respectively.
Figure 5. Schematic representation of periodic one-dimensional magnonic crystals surrounded by a third ferromagnetic material considered to investigate spin wave transmission.

To the periodic superlattice shown in figure 3(a), equation (8) applied at the interface \( A|B \) relates the amplitudes \( (A_l, A'_l) \) with \( (B_l, B'_l) \), while the same equation when applied at interface \( B|A \) relates the amplitudes \( (A_{l+1}, A'_{l+1}) \) with \( (B_l, B'_l) \). These equations can be written as

\[
M_A \begin{pmatrix} A_l \\ A'_l \end{pmatrix} = N_B \begin{pmatrix} B_l \\ B'_l \end{pmatrix} \quad (11)
\]

and

\[
M_B \begin{pmatrix} B_l \\ B'_l \end{pmatrix} = N_A \begin{pmatrix} A_{l+1} \\ A'_{l+1} \end{pmatrix}, \quad (12)
\]

which can be easily combined to obtain

\[
\begin{pmatrix} A_{l+1} \\ A'_{l+1} \end{pmatrix} = N_A^{-1} M_B N_B^{-1} M_A \begin{pmatrix} A_l \\ A'_l \end{pmatrix}. \quad (13)
\]

By using Bloch’s theorem, we can obtain

\[
\cos(QD) = \left( \frac{1}{2} \right) \text{Tr} [T] \quad (14)
\]

with \( T = N_A^{-1} M_B N_B^{-1} M_A \). The form of the \( M_s \) and \( N_s \) can be found in [41]. The matrix \( T \) is called a transfer matrix because it relates the SW amplitudes of the \((l + 1)\)th unit cell to the amplitudes in \( l \)th unit cell. The last line follows from the fact that \( T \) is a unimodular \( 2 \times 2 \) matrix. Equation (14) describes the bulk modes of SWs in a periodic arrangement of the magnetic multilayers.

Once we know the form of the transfer-matrix \( T \), the bulk SW spectra are determined. If we have an \( A|A \) (or \( B|B \)) interface, we just need to change the interface exchange constant from \( I \) to \( I_{A|B} \). Another important result is that this expression also holds for any other arrangement of the magnetic multilayers, and therefore will be employed in the determination of the spectra for the quasiperiodic structures.

2.2. Transfer-matrix method for spin wave transmission

In order to calculate the SWs that can propagate in one-dimensional magnonic crystals, we consider that the periodic structure MC (we suppose that this structure is repeated \( N \) times) shown in figure 3(a) is surrounded by a third simple cubic spin-\( S \) Heisenberg semi-infinite ferromagnetic material \( C \), with average spin \( S_C \), exchange term \( J_C \) and lattice constant \( a \), as is shown in the schematic diagram in figure 5. The exchange term at the interface \( \alpha|\beta \) is \( I_{\alpha\beta} \) (\( \alpha \) and \( \beta \) being \( A \), \( B \) or \( C \)). As commented in section 2.1, to determine the spin wave transmittance we must solve equation (8) in \( A \), \( B \) and \( C \), considering the solution proposed in equation (10), changing the amplitudes and indices \( A \) by \( C \), and taking \( r_C \) as the origin of the \( C \) slab at the interface.

The boundary conditions are written in a matrix form which can be represented, after some straightforward algebra, as

\[
\begin{pmatrix} C_0 \\ C'_0 \end{pmatrix} = M_C M_A M_{AB} M_B \ldots M_{AB} M_B M_{BC} \begin{pmatrix} C_{N+1} \\ C'_{N+1} \end{pmatrix} = M_N \begin{pmatrix} C_{N+1} \\ 0 \end{pmatrix}. \quad (15)
\]

Here \( M_N \) is the transfer-matrix associated with the \( N \) repetitions of the periodic multilayer under consideration. It connects the SW amplitudes of the upper semi-infinite slab \( (C_0, C'_0) \) to the spin wave amplitudes of the lower semi-infinite slab \( (C_{N+1}, 0) \). \( M_N \) is the result of multiplication of two \( 2 \times 2 \) matrices: the transmission matrix \( M_{AB} \), which
represents the transmission of a normal incident SW across interfaces $\alpha \rightarrow \beta$ ($\alpha$ and $\beta$ being $A$, $B$ or $C$), and the propagation matrix $M_{\alpha\beta}$, which represents the propagation of the SW inside a certain slab $\gamma$ ($\gamma = A$ or $B$). These two matrices are given by [38]

$$M_{\alpha\beta} = \frac{I_{\alpha\beta} - \hat{\lambda}_{\alpha\beta}}{I_{\alpha\beta} - \hat{\lambda}_{\alpha\beta}} \cdot \frac{(I_{\alpha\beta} - \hat{\lambda}_{\alpha\beta})}{(I_{\alpha\beta} - \hat{\lambda}_{\alpha\beta})}.$$

and

$$M_{\gamma} = \begin{bmatrix} t_{\gamma} & 0 \\ 0 & t_{\gamma}^{-1} \end{bmatrix},$$

where we have defined

$$\lambda_{\alpha\beta} = -\omega + I_{\alpha\beta} S_{\beta} + J_{\alpha} S_{\alpha} [5 - \gamma]_{\alpha} - \hat{\alpha}_{\gamma},$$

$$\hat{\lambda}_{\alpha\beta} = -\omega + I_{\alpha\beta} S_{\beta} + J_{\alpha} S_{\alpha} [5 - \gamma]_{\alpha} - \hat{\alpha}_{\gamma},$$

and

$$f_{\alpha} = e^{ik_{\alpha}} = t_{\alpha} e^{ik_{\alpha} n_{\alpha}} = (f_{\alpha})^{n_{\alpha}}.$$

The terms $\hat{\alpha}_{\gamma}$ and $\tilde{\alpha}_{\gamma}$ are the complex conjugate of $f_{\alpha}$ and $t_{\alpha}$, respectively. $M_{\alpha\beta}$ and $M_{\gamma}$ are completely similar to the matrices found in the work of Vasconcelos and Albuquerque [39].

Once we know the transfer-matrix $M_{\gamma}$ from equation (15), we have that the transmittance is simply given by

$$T = \left| C_{N+1} / C_0 \right|^2 = \left| \frac{1}{M_{\gamma}(1,1)} \right|^2.$$

In addition, the reflectance is related to the transfer-matrix by

$$R = \left| C_0 / C_0 \right|^2 = \left| \frac{M_{\gamma}(2,1)}{M_{\gamma}(1,1)} \right|^2.$$

We have obtained the expressions that determine the dispersion relation and transmission, namely, equations (14) and (21), respectively, and we are now able to obtain the transfer-matrices for the quasiperiodic generalized Fibonacci sequences.

3. Transfer-matrices for quasiperiodic generalized Fibonacci sequences

We now intend to obtain a general transfer-matrix expression for the dispersion relation and transmittance of SWs in MQCs arranged in accordance with GFs. A generalized Fibonacci structure, as discussed in section 1, can be experimentally grown by juxtaposing two building blocks $A$ and $B$ (corresponding to slabs $A$ and $B$), in such a way that the $n$th stage of the sequence $S_n$ is given iteratively by the rule $S_n = S_{n-1} S_{n-2}$ ($n \geq 2$), with $S_0 = B$ and $S_1 = A$. This recurrence rule is also invariant under the transformations $A \rightarrow AB$ and $B \rightarrow A$.

Firstly, let us analyze the transfer-matrix method for SW dispersion. For any values of $p$ and $q$, it is easy to show that the transfer-matrix for the $n$th generation of the sequence $\sigma(p, q)$ is given by [42]

$$T_{\sigma_n} = (T_{\sigma_{n-1}})^{(1)}(T_{\sigma_{n-1}})^{(2)},$$

$$\sigma_n = (T_{\sigma_{n-2}})^{(1)}(T_{\sigma_{n-2}})^{(2)}.$$

where $(T_{\sigma_n})^{p(q)}$ means that matrix $T_{\sigma_n}$ is multiplied $p(q)$ times. Therefore, from knowledge of the transfer-matrices $T_{\sigma_1}$ and $T_{\sigma_2}$ we can determine the transfer-matrix of any generation, which are given by

$$T_{\sigma_1} = N_{AA}^{-1} M_{AA},$$

and

$$T_{\sigma_2} = N_{A}^{-1} M_{B} \left( N_{BB}^{-1} M_{BB} \right)^{(q-1)} N_{B}^{-1} M_{A} \times \left( N_{AA}^{-1} M_{AA} \right)^{(p-1)}.$$

For the case of SW transmission in 1D MQCs, the general formula for a transfer-matrix is

$$M_{n}^N = \begin{cases} M_{CA} (T_n)^{N-1} T_n M_{BC} & \text{(even)} \\ M_{CA} (T_n)^{N-1} T_n M_{MC} & \text{(odd)} \end{cases}$$

with

$$T_n = \begin{cases} (T_{n-1} M_{AA})^p (T_{n-2} M_{BA})^q & \text{(even)} \\ (T_{n-1} M_{BA})^p (T_{n-2} M_{AA})^q & \text{(odd)} \end{cases}$$

and

$$T_2 = (M_{AA} M_{MA} M_{AB} M_{BB})^q M_{B}.$$
2, \( I_A = I/J_A = 1.2 \), \( I_B = I/J_B = 2.4 \), \( I_{AA} = I_A/J_A = I_{BB} = I_B/J_B = 1 \), \( H_{AA} = H_{BB} = g \mu_B H_0/J_B = 0 \). Here, we also take \( k_A = k_B = 0 \). For simplicity, we chose the parameters of medium \( C \) as being the same of those of \( A \), i.e. we make \( C = A \). This will be very important for understanding the figures below; however, this does not affect qualitatively the transmittance spectra of SWs.

In figure 6, we show the plot of dispersion relation (left) and transmission (right) of SWs in a MQC built up in accordance with the GM Fibonacci arrangement for second (figure 6(a)) and third (figure 6(b)) generations, respectively. The second generation of this sequence corresponds to a periodic MC. Looking to figure 6(a), we observe that the relation dispersion displays a total of eight allowed branches (for the case in which \( n_A = n_B = 4 \), the number of allowed modes is related to the number of layers in each material, and it increases with \( 4 \times F_n \), one for each monolayer [43]: six at \( 0 \leq \Omega \leq 3 \) (black lines), and two at \( \Omega \geq 4 \) (red lines). As was discussed in section 2.1, for \( \Omega \geq 4 \), both \( k_A \) and \( k_B \) assume complex values. Nevertheless, the Bloch wavevector \( Q \), which is the wavevector of the superlattice, can assume real values, but these allowed bands are very narrow. Comparing the dispersion and transmission plots, we can see a perfect correspondence between them only for \( 0 \leq \Omega \leq 3 \). There is no transmission corresponding to the two upper modes. This occurs because the SW is dependent on the propagation medium, and as the limit of the highest frequency is in the medium \( A \) (see figure 4), where \( \Omega_{\text{max}} = 4 \) (as we said before, we consider the medium \( C \) equal to the medium \( A \)), we will not have transmission for \( \Omega > 4 \).

In figure 6(b), which represents a small modification in a unit cell of the periodic case \( (S_2 = [A|B] \) and \( S_3 = [A|B|A] \)), we can see that more allowed bands appear, as expected, but some these branches (blue lines) are in a frequency region \( (3 < \Omega < 4) \) which corresponds to magnonic band gaps for the periodic case. This is a very important and interesting characteristic of quasiperiodic systems: allowed modes emerge in regions which before were band gap ones without the need to insert defects, as is done in doped semiconductors [19].

The green dashed boxes in figures 6(a) and (b) show us regions in which the SW transmission spectra present a mirror symmetry around a given frequency, called the mid-gap frequency (MGF). This behavior is very common in light transmission in dielectric multilayered structures (like Fabry–Perot interferometers) when the quarter-wavelength condition is satisfied, i.e. \( nd = \lambda_0/4 \), where \( n \) and \( d \) are the refractive index and the thickness of the material, respectively. This condition can be easily satisfied in such materials because the light relation dispersion is linear \( (\omega \propto k) \). However, for MCs the relation dispersion has the form \( \omega = (ka)^2 \), for \( ka < 1 \) (see equation (9)). Even so, the MC transmission spectra reveal that there must be a way to engineer a multilayered MC with parameters previously determined so it can obey completely the quarter-wavelength condition.

In figures 7 and 8 we present the same plot as in figure 6, but for SM and BM Fibonacci sequences, respectively; figures with the label (a) correspond to the second generation while those labeled (b) correspond to the third generation. In these quasiperiodic sequences, there are more \( A \) slabs than \( B \) ones. We can observe that very rich and interesting magnonic pass- and stop-band structures emerge in these quasiperiodic systems. Here we have a pass- and stop-band structure more interesting than GM case as a consequence of disorder, yet it is still deterministic. However, these new allowed or forbidden bands are distributed in range \( 0 \leq \Omega \leq 4 \). Observe yet bands formed in \( 0 \leq \Omega \leq 3 \) always are wider than ones formed in \( 3 < \Omega \leq 4 \). As occurred with the GM sequence, in the SM and BM cases we also have green dashed boxes in figures 6(a) and (b) which shows regions in which the SW transmission spectra present a mirror symmetry of the transmission spectra around a MGF. Another important characteristic of the GM, SM and BM magnonic nanostructures, shown in a previous work [42], is that the SW spectra are fractals.
Figure 7. The same as figure 6 but for the SM Fibonacci case. In (a) we have the second generation with \( N = 25 \), while in (b) the spectra correspond to the third generation and \( N = 11 \).

Finally, we present the SW spectra for MQCs in accordance with CM and NM Fibonacci sequences in figures 9 and 10, respectively. The labels (a) and (b) correspond to second and third generations, respectively. Unlike the cases discussed above, in CM and NM MQCs there are more \( B \) slabs than \( A \) ones. This fact implies that most of the allowed bands appear in the frequency region \( 0 \leq \Omega \leq 3 \), and that the modes localized in region \( 3 < \Omega \leq 4 \) are very narrow. Here, the spectra also reveal a rich and interesting magnonic pass- and stop-band structures. The transmission spectra in CM and NM cases has a mirror symmetry shown in the green dashed boxes. However, these spectra do not present a fractal behavior as seen in the previous cases.

5. Conclusions

In summary, we have employed a microscopic theory to investigate propagation of SWs through their dispersion and transmission spectra in systems arranged so as to display deterministic disorder. Here, the quasiperiodic arrangement investigated is the well-known GFS, which is characterized by the \( \sigma(p, q) \) parameter, where \( p \) and \( q \) are non-zero integers. In our numerical results we note that there is no transmission corresponding to the two upper modes if we consider the periodic case (see figure 6(a)). This occurs because we made choice that media \( A \) and \( C \) are made of the same material. So, the transmission vanishes for the region in which \( \Omega > 4 \). This led us to choose, in other figures, the frequency axis only for \( 0 \leq \Omega \leq 4 \) (see figure 4 and the comment about it).

Another interesting conclusion is that when we consider the allowed bands, not all modes have the same probability of transmission, i.e. the modes are more likely to be transmitted in the system at frequencies closer to the edges of the allowed bands. When we consider the Fibonacci generalization, the SW spectra reveal a rich and interesting magnonic pass- and stop-band structure, including a almost symmetric band gap distribution around of a mid-gap frequency (which varies with the GFSs). Also, the transmission spectra in CM and NM cases do not present a fractal behavior because \( \sigma \) is not a
Figure 9. The same as figure 6, but for the CM Fibonacci case. In (a) we have the second generation with $N = 25$, while in (b) the spectra correspond to the third generation and $N = 16$.

PV-type number, i.e. the fluctuation of the physical properties of these substitution sequence is more accentuated.

All physical phenomena presented here can be tested experimentally. A good example is the inelastic light scattering spectroscopy of Raman and Brillouin type. Techniques involving magnetic resonance (for example, ferromagnetic resonance, standing SW resonance, etc) can also be used, and, indeed, they have previously been successfully applied to surface and bulk SWs in various magnetic microstructures [28, 40]. Other experimental techniques, like grating coupling and attenuated total reflection, were proposed by a number of authors (for a review see [40]). Finally, we hope that our theoretical results can stimulate experimental groups to prove them.

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