

LOCALIZATION OF A MOBILE ROBOT BASED IN ODOMETRY AND NATURAL LANDMARKS USING EXTENDED KALMAN FILTER

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Keywords: Robot Localization, Kalman Filter, Sensor Fusion.

Abstract: This work proposes a localization system for mobile robots using the Extended Kalman Filter. The robot navigates in an known environment where the lines of the floor are used as natural landmarks and identified by using the Hough transform. The prediction phase of the Kalman Filter is implemented using the odometry model of the robot. The update phase directly uses the parameters of the lines detected by the Hough algorithm to correct the robot's pose.

1 INTRODUCTION

Borenstein et al. have classified the localization methods in two great categories: relative localization methods, which give the robot's pose relative to the initial one, and absolute localization methods, which indicate the global pose of the robot and do not need previously calculated poses (Borenstein et al., 1997).

As what concerns wheel robots, it is common the use of encoders linked to wheel rotation axes, a technique which is known as odometry (Borenstein et al., 1997). However, the basic idea of odometry is the integration of the mobile information in a determined period of time, what leads to the accumulation of errors (Park et al., 1998).

The techniques of absolute localization use landmarks to locate the robot. These landmarks can be artificial ones, when introduced in the environment aiming at assisting at the localization of the robot, or natural ones, when they can be found in the proper environment. It is important to underline that even the techniques of absolute localization are inaccurate due to noises produced by the manipulated sensors.

Literature shows works using distance measures to natural landmarks (walls, for example) to locate the robot. The obtaining of these measures is generally made with the help of sonar, laser and computational vision (Lizzaralde et al., 2003; Kim and Kim, 2004; Pres et al., 1999).

Bezerra used in his work the lines of the floor composing the environment as natural landmarks (Bezerra, 2004). Kiriy and Buehler, have used extended Kalman Filter to follow a number of artificial landmarks placed in a non-structured way (Kiriy and Buehler, 2002). Launay et al. employed ceiling lamps of a corridor to locate the robot (Launay et al., 2002).

This paper proposes a system enabling to locate a mobile robot in an environment in which the lines of the floor form a bi-dimensional grid. To turn it possible, the lines are identified as natural landmarks and its characteristics, as well as the odometry model of the robot, are incorporated in a Kalman Filter in order to get its pose.

2 THE KALMAN FILTER

The modeling of the Discrete Kalman Filter - DKF presupposes that the system is linear and described by the model of the equations of the system (1):

$$\begin{cases} \mathbf{s}_t = \mathbf{A}_t \mathbf{s}_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1} + \gamma_t \\ \mathbf{z}_t = \mathbf{C}_t \mathbf{s}_t + \delta_t \end{cases} \quad (1)$$

in which $\mathbf{s} \in R^n$ is the vector of the states; $\mathbf{u} \in R^l$ is the vector of the control entrances; $\mathbf{z} \in R^m$ is the vector of measurements; the matrix $n \times n$, \mathbf{A} , is the transition

matrix of the states; \mathbf{B} , $n \times l$, is the coefficient matrix on entry; matrix \mathbf{C} , $m \times n$, is the observation matrix; $\gamma \in R^n$ represents the vector of the noises to the process and $\delta \in R^m$ the vector of measurement errors. Indexes t and $t - 1$ represent the present and the previous instants of time.

The Filter operates in prediction-actualization mode, taking into account the statistical proprieties of noise. An internal model of the system is used to updating, while a retro-alimentation scheme accomplishes the measurements. The phases of prediction and actualization to DKF can be described by the systems (2) and (3) respectively.

$$\begin{cases} \bar{\mu}_t = \mathbf{A}_t \mu_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1} \\ \bar{\Sigma}_t = \mathbf{A}_t \Sigma_{t-1} \mathbf{A}_t^T + \mathbf{R}_t \end{cases} \quad (2)$$

$$\begin{cases} \mathbf{K}_t = \bar{\Sigma}_t \mathbf{C}_t^T (\mathbf{C}_t \bar{\Sigma}_t \mathbf{C}_t^T + \mathbf{Q}_t)^{-1} \\ \mu_t = \bar{\mu}_t + \mathbf{K}_t (\mathbf{z}_t - \mathbf{C}_t \bar{\mu}_t) \\ \Sigma_t = (\mathbf{I} - \mathbf{K}_t \mathbf{C}_t) \bar{\Sigma}_t \end{cases} \quad (3)$$

The Kalman Filter represents the vector of the states \mathbf{s}_t by its mean μ_t and co-variance Σ_t . Matrixes \mathbf{R} , $n \times n$, and \mathbf{Q} , $l \times l$, are the matrixes of the co-variance of the noises of the process (γ) and measurement (δ) respectively, and matrix \mathbf{K} , $n \times m$, represents the prot of the system.

A derivation of the Kalman Filter applied to non-linear systems is the Extended Kalman Filter - EKF. The idea of the EKF is to linearize the functions around the current estimation using the partial derivatives of the process and of the measuring functions to calculate the estimations, even in the face of non-linear relations. The model of the system to EKF is given by the system (4):

$$\begin{cases} \mathbf{s}_t = g(\mathbf{u}_{t-1}, \mathbf{s}_{t-1}) + \gamma_t \\ \mathbf{z}_t = h(\mathbf{s}_t) + \delta_t \end{cases} \quad (4)$$

in which $g(\mathbf{u}_{t-1}, \mathbf{s}_{t-1})$ is a non-linear function representing the model of the system, and $h(\mathbf{s}_t)$ is a non-linear function representing the model of the measurements.

Their prediction and actualization phases can be obtained by the systems of equations (5) and (6) respectively.

$$\begin{cases} \bar{\mu}_t = g(\mathbf{u}_{t-1}, \mu_{t-1}) \\ \bar{\Sigma}_t = \mathbf{G}_t \Sigma_{t-1} \mathbf{G}_t^T + \mathbf{R}_t \end{cases} \quad (5)$$

$$\begin{cases} \mathbf{K}_t = \bar{\Sigma}_t \mathbf{H}_t^T (\mathbf{H}_t \bar{\Sigma}_t \mathbf{H}_t^T + \mathbf{Q}_t)^{-1} \\ \mu_t = \bar{\mu}_t + \mathbf{K}_t (\mathbf{z}_t - h(\bar{\mu}_t)) \\ \Sigma_t = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t) \bar{\Sigma}_t \end{cases} \quad (6)$$

The matrix \mathbf{G} , $n \times n$, is the jacobian term linearizes the model and \mathbf{H} , $l \times n$ is the jacobian term linearizes

the measuring vector. Such matrixes are defined by the equations (7) e (8).

$$\mathbf{G}_t = \frac{\partial g(\mathbf{u}_{t-1}, \mathbf{s}_{t-1})}{\partial \mathbf{s}_{t-1}} \quad (7)$$

$$\mathbf{H}_t = \frac{\partial h(\mathbf{s}_t)}{\partial \mathbf{s}_t} \quad (8)$$

Next we will describe the modeling of the problem, as well as the definition of the matrixes which will be employed in the Kalman Filter.

3 MODELING

3.1 Prediction phase: Odometer model of the robots movement

A classic method used to calculate the pose of a robot is the odometry. This method uses sensors, optical encoders, for example, which measure the rotation of the robot's wheels. Using the cinematic model of the robot, its pose is calculated by means of the integration of its movements from a referential axis.

As encoders are sensors, normally their reading would be implemented in the actualization phase of the Kalman Filter, not in the prediction phase. Thrun et al. propose that odometer information does not function as sensorial measurements; rather they suggest incorporating them to the robot's model (Thrun et al., 2005).

In order that this proposal is implemented, one must use a robot's cinematic model considering the angular displacements of the wheels as signal that the system is entering in the prediction phase of the Kalman Filter.

Consider a robot with differential drive in which the control signals applied and its actuators are not tension, instead angular displacement, according to Figure 1.

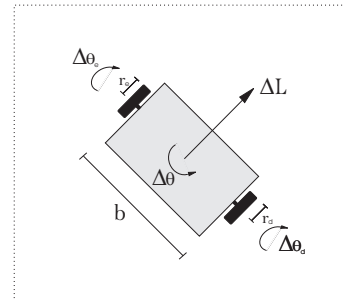


Figure 1: Variables of the geometric model.

With this idea, and supposing that speeds are constant in the sampling period, one can determine the geometric model of the robot's movement(system 9).

$$\begin{cases} x_t = x_{t-1} + \\ \frac{\Delta L}{\Delta \theta} [\sin(\theta_{t-1} + \Delta \theta) - \sin(\theta_{t-1})] \\ y_t = y_{t-1} - \\ \frac{\Delta L}{\Delta \theta} [\cos(\theta_{t-1} + \Delta \theta) - \cos(\theta_{t-1})] \\ \theta_t = \theta_{t-1} + \Delta \theta \end{cases} \quad (9)$$

The turn easier the readability of the system (9) representing the odometry model of the robot, two auxiliary variables have been employed ΔL and $\Delta \theta$

$$\begin{cases} \Delta L = (\Delta \theta_d r_d + \Delta \theta_e r_e) / 2 \\ \Delta \theta = (\Delta \theta_d r_d - \Delta \theta_e r_e) / b \end{cases} \quad (10)$$

in which $\Delta \theta_d$ is the reading of the right encoder and functions relatively the robot by means of the angular displacement of the right wheel; $\Delta \theta_e$ is the reading of the left encoder and functions as a displacement applied to the left wheel; b represents the distance from wheel to wheel of the robot; r_d and r_e are the spokes of the right and the left wheels respectively.

It is important to emphasize that in real applications the angular displacement effectively realized by the right wheel differs of that measured by the encoder. Besides that, the supposition that the speeds are constant in the sampling period, which has been used to obtain the model 9, is not always true. Hence, there are differences between the " angular displacements of the wheels ($\Delta \hat{\theta}_d$ e $\Delta \hat{\theta}_e$) and those ones measured by the encoders ($\Delta \theta_d$ e $\Delta \theta_e$). This difference will be modeled by a Gaussian noise, according to system (11).

$$\begin{cases} \Delta \hat{\theta}_d = \Delta \theta_d + \varepsilon_d \\ \Delta \hat{\theta}_e = \Delta \theta_e + \varepsilon_e \end{cases} \quad (11)$$

It is known that odometry possesses accumulative error. Therefore, the noises ε_d and ε_e do not possess constant variance. It is presumed that these noises present a proportional standard deviation to the module of the measured displacement.

With these new considerations, system (9) is now represented by system (12):

$$\begin{cases} x_t = x_{t-1} + \\ \frac{\Delta \hat{L}}{\Delta \hat{\theta}} [\sin(\theta_{t-1} + \Delta \hat{\theta}) - \sin(\theta_{t-1})] \\ y_t = y_{t-1} - \\ \frac{\Delta \hat{L}}{\Delta \hat{\theta}} [\cos(\theta_{t-1} + \Delta \hat{\theta}) - \cos(\theta_{t-1})] \\ \theta_t = \theta_{t-1} + \Delta \hat{\theta} \end{cases} \quad (12)$$

in which

$$\begin{cases} \Delta \hat{L} = (\Delta \hat{\theta}_d r_d + \Delta \hat{\theta}_e r_e) / 2 \\ \Delta \hat{\theta} = (\Delta \hat{\theta}_d r_d - \Delta \hat{\theta}_e r_e) / b \end{cases} \quad (13)$$

One should observe that this model can not be used when $\Delta \hat{\theta} = 0$. When it occurs, one uses an odometry module simpler than a robot (system 14), obtained from the limit of system 12 when $\Delta \hat{\theta} \rightarrow 0$.

$$\begin{cases} x_t = x_{t-1} + \Delta \hat{L} \cos(\theta_{t-1}) \\ y_t = y_{t-1} + \Delta \hat{L} \sin(\theta_{t-1}) \\ \theta_t = \theta_{t-1} \end{cases} \quad (14)$$

Thrun's idea implies a difference as what concerns system (4), because the noise is not audible; rather, it is incorporated to the function which describes the model, as system (15) shows:

$$\begin{cases} \mathbf{s}_t = p(\mathbf{u}_{t-1}, \mathbf{s}_{t-1}, \boldsymbol{\varepsilon}_t) \\ \mathbf{z}_t = h(\mathbf{s}_t) + \boldsymbol{\delta}_t \end{cases} \quad (15)$$

in which $\boldsymbol{\varepsilon}_t = [\varepsilon_d \ \varepsilon_e]^T$ is the noise vector connected to odometry.

It is necessary, however, to bring about a change in the prediction phase of the system (6) resulting in the system (16) equations:

$$\begin{cases} \bar{\boldsymbol{\mu}}_t = \boldsymbol{\mu}_{t-1} + p(\mathbf{u}_{t-1}, \boldsymbol{\mu}_{t-1}, 0) \\ \bar{\boldsymbol{\Sigma}}_t = \mathbf{G}_t \boldsymbol{\Sigma}_{t-1} \mathbf{G}_t^T + \mathbf{V}_t \mathbf{M}_t \mathbf{V}_t^T \end{cases} \quad (16)$$

in which \mathbf{M} , $l \times l$, is the co-variance matrix of the noise sensors ($\boldsymbol{\varepsilon}$) and \mathbf{V} , $n \times m$, is the jacobian mapping the sensor noise to the space of state. Matrix \mathbf{V} is defined by the equation (17).

$$\mathbf{V}_t = \frac{\partial p(\mathbf{u}_{t-1}, \mathbf{s}_{t-1}, 0)}{\partial \mathbf{u}_{t-1}} \quad (17)$$

Making use of the odometry model of the robot described in this section and the definitions of the ma-

trices used by the Kalman Filter, we have:

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & g_{13} \\ 0 & 1 & g_{23} \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{onde} \quad (18)$$

$$g_{13} = \frac{\Delta \hat{L}}{\Delta \hat{\theta}} [\cos(\theta_{t-1} + \Delta \hat{\theta}) - \cos(\theta_{t-1})]$$

$$g_{23} = \frac{\Delta \hat{L}}{\Delta \hat{\theta}} [\sin(\theta_{t-1} + \Delta \hat{\theta}) - \sin(\theta_{t-1})]$$

$$\mathbf{V} = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \\ r_d/b & -r_e/b \end{pmatrix}, \quad \text{onde} \quad (20)$$

$$v_{11} = k1 \cos(k2) - k3 [\sin(k2) - \sin(\theta_{t-1})]$$

$$v_{12} = -k1 \cos(k2) + k3 [\sin(k2) - \sin(\theta_{t-1})]$$

$$v_{21} = k1 \sin(k2) - k3 [-\cos(k2) + \cos(\theta_{t-1})]$$

$$v_{22} = -k1 \sin(k2) + k3 [-\cos(k2) + \cos(\theta_{t-1})]$$

$$\mathbf{M} = \begin{pmatrix} (\alpha_1 |\Delta \hat{\theta}_d|)^2 & 0 \\ 0 & (\alpha_2 |\Delta \hat{\theta}_e|)^2 \end{pmatrix} \quad (22)$$

Elements m_{11} and m_{22} in the equation (22) represent the fact that the standard deviations of ϵ_d and ϵ_e are proportional to the module of the angular displacement. The variables $k1$, $k2$ and $k3$ are given by system (23), considering $r_d = r_e = r$.

$$\begin{cases} k1 = \frac{r(\Delta \hat{\theta}_d + \Delta \hat{\theta}_e)}{b(\Delta \hat{\theta}_d - \Delta \hat{\theta}_e)} \\ k2 = \theta_{t-1} + \frac{r(\Delta \hat{\theta}_d - \Delta \hat{\theta}_e)}{b} \\ k3 = \frac{b\Delta \hat{\theta}_e}{2(r(\Delta \hat{\theta}_d - \Delta \hat{\theta}_e)/b)^2} \end{cases} \quad (23)$$

3.2 Update Phase: Sensor Model for Detecting Natural Landmarks

In this work we will use as natural landmarks a set of straight lines formed by the grooves of the floor in the environment where the robot will navigate because, besides being already available in the referred environment, they are also very common in the real world.

Due to the choice of the straight lines as landmarks, the technique adopted to identify them was the Hough transform. This kind of transform is a method employed to identify inside a digital image a class of

geometric forms which can be represented by a parametric curve (Gonzales, 2000). As what concerns the straight lines, a mapping is provided between the Cartesian space (X, Y) and the space of the parameters (ρ, α) where the straight line is defined.

Hough defines a straight line using its common representation, as equation (24) shows, in which parameter ρ represents the length of the vector and α the angle this vector forms with axis X. Figure 2 shows the geometric representation of these parameters.

$$\rho = x \cos(\alpha) + y \sin(\alpha) \quad (24)$$

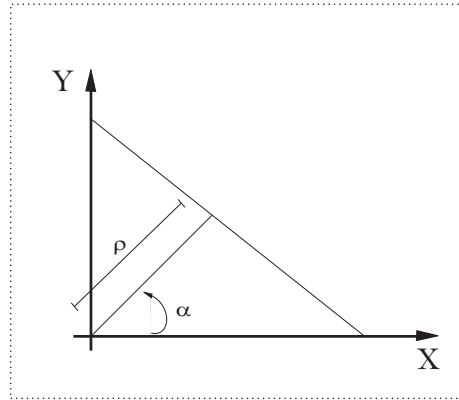


Figure 2: Parameters of the Hough.

The system discussed in this paper is based in a robot with differential drive, possessing a rm and stable camera embedded in its structure, as in Figure 3. The idea is to use information obtained directly on the image processing (ρ, α) in the actualization phase of an Extended Kalman Filter to calculate the robot's pose. Thereof, one must deduct the sensor model (that is, the image processor) in function of the variables of state.

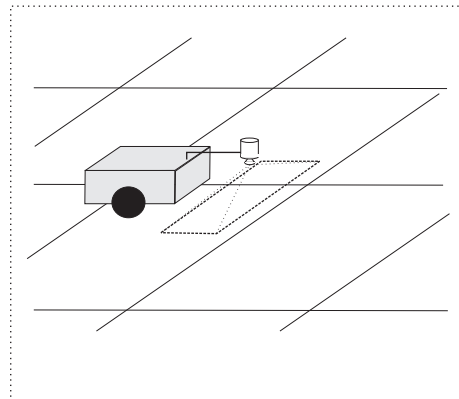


Figure 3: Robotic system.

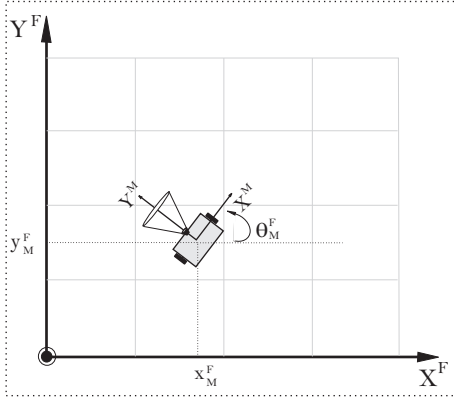


Figure 4: System of Coordinates.

Robot navigates an environment in which the position of the straight lines of the world (α^F , ρ^F) is known and, at each step, it identifies the descriptors of the straight lines contained in the image (α^M , ρ^M) using image processing and the parameters of the camera gauging.

Figure 4 illustrates the systems of steady {F} and mobile {M} coordination used to mathematic deduction of the sensor model. The point (x_M^F, y_M^F) is the coordinate of origin of the mobile system mapped in the system of steady and mobile coordinates, while variable θ_M^F represents the rotation angle of the system of mobile coordinates.

Our point of departure is a simple change mapping a point in the system de mobile {M} coordinates for the system of steady {F} coordinates, as in system (25).

$$\begin{cases} x^F = \cos(\theta_M^F)x^M - \sin(\theta_M^F)y^M + x_M^F \\ y^F = \sin(\theta_M^F)x^M + \cos(\theta_M^F)y^M + y_M^F \end{cases} \quad (25)$$

Using an equation (24) and considering the system of steady {F} coordinates, we have:

$$\rho^F = x^F \cos(\alpha^F) + y^F \sin(\alpha^F) \quad (26)$$

Also using the definition of system (24), but now considering the system of mobile {M} coordinates, we have:

$$\rho^M = x^M \cos(\alpha^M) + y^M \sin(\alpha^M) \quad (27)$$

Replacing (25) in (26) and doing the necessary equivalences with system (27), we can obtain system (28), which represents the sensor module to be used in the filter.

$$\begin{cases} \alpha^M = \alpha^F - \theta_M^F \\ \rho^M = \rho^F - x_M^F \cos(\alpha^F) - y_M^F \sin(\alpha^F) \end{cases} \quad (28)$$

In this system, α^F and ρ^F are given, because they represent the description of the map landmark, which is supposedly known. The equations express the relations among the returned information (α^F , ρ^F) and the height that one wants to estimate (x_F^M, y_F^M, θ_F^M).

One should note that there is a straight relation among these variables (x_F^M, y_F^M, θ_F^M) and the robot's pose (x_R, y_R, θ_R) , which is given by system 29.

$$\begin{cases} x_R = x_F^M \\ y_R = y_F^M \\ \theta_R = \theta_F^M + \frac{\pi}{2} \end{cases} \quad (29)$$

The model of system 28 is incorporated to the Kalman Filter through matrix H (equation 8), given by equation 30.

$$H = \begin{pmatrix} -\cos(\theta_F^M) & -\sin(\theta_F^M) & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (30)$$

4 RESULTS

The situations presented here have been obtained by simulation. We tried to use the noise measure of the sensors consistent to reality. For that, it has been embedded to encoders a noise which standard deviation is proportional to the amount of read pulses. In the identification of the parameters of the straight lines ρ and α , the standard deviation of noise also obeys a proportion which is ruled by the size that the straight line occupies in the image.

In the Figures, the hatched rectangle represents the robot's real pose, while the continuous rectangle, the calculated pose.

Figure 5 presents the result of the localization system using only odometry.

Another localization system largely used has also been implemented: the localization system using geometric correction. In this system, at each step the straight lines are identified and used to calculate the robot's pose using trigonometry. When there are no straight lines, the correction is made by odometry (Figure 6).

Finally, in Figure 7 is shown the result of the pose calculation using the fusion of the data of odometry and of the landmark detection by EKF.

A particular situation has been implemented to test the robustness of the localization systems. For that, it has been introduced to the system a perturbation whenever the robot gets near position (6.5,4.5). The results coming from the use of geometry and Kalman are exhibited in Figures 8 and 9.

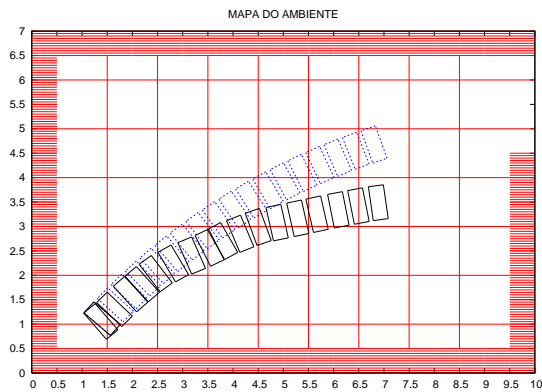


Figure 5: Localization by odometry.

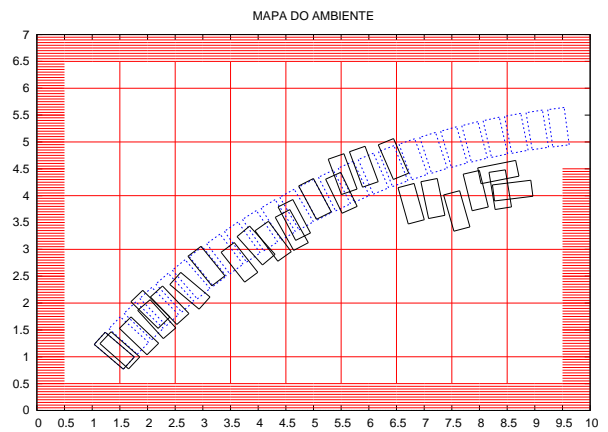


Figure 8: Effect of perturbation: Localization by odometry and geometric correction.

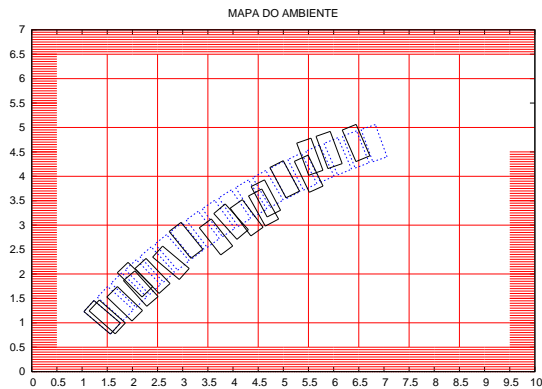


Figure 6: Localization by odometry and geometric correction.

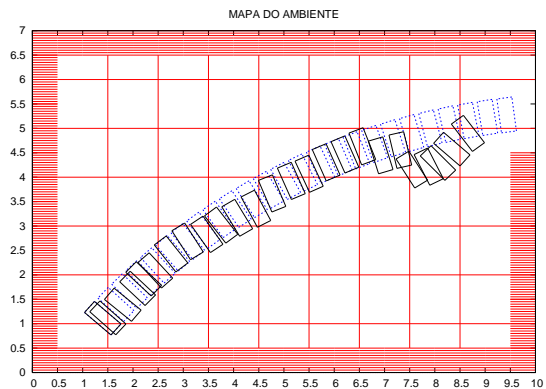


Figure 9: Effect of perturbation: Localization using Extended Kalman Filter.

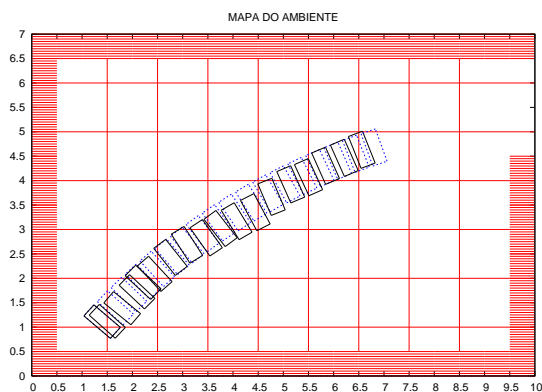


Figure 7: Localization using Extended Kalman Filter.

5 CONCLUSIONS AND PERSPECTIVES

This paper has proposed a localization system for mobile robots using the Extended Kalman Filter. The main contribution is the modeling of the optic sensor made such a way that it permits using the parameters obtained in the image processing directly to equations of the Kalman Filter, without passing any intermediate phase of pose calculation, or of distance, only with the available usual information.

When analyzing Figures 5, 6 and 7 one can perceive that the behavior of the localization system using the Kalman Filter has proved more satisfactory than those using odometry and geometric corrections.

As what concerns perturbation rejections (Figures 8 e 9) the system based in Kalman also proved efficient, for it tends to return to real pose, while the system based in geometric correction did not exhibit

the same performance.

As future works, we intend to: Implement other formulations of the Kalman Filter. For example, the Kalman Filter with Partial Observations; Replace the Kalman Filter by a Filter of Particles, having in view that the latter incorporates more easily the non-linearities of the problem, besides leading with non-Gaussian noises; Develop this strategy of localization to a proposal of SLAM (Simultaneous Localization and Mapping), so much that robot is able of doing its localization without a previous knowledge of the map and, simultaneously, mapping the environment it navigates.

ACKNOWLEDGEMENTS

We thanks CAPES and CNPq by the financial support.

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