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Effective potential in the boundary effective theory formalism

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We calculate the one-loop effective potential at finite temperature for a system of massless scalar fields with quartic interaction $\lambda\phi^4$ in the framework of the boundary effective theory formalism. The calculation relies on the solution of the classical equation of motion for the field and Gaussian fluctuations around it. Our result is nonperturbative and differs from the standard one-loop effective potential for field values larger than $T/\sqrt{\lambda}$.

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I. INTRODUCTION

In thermal systems containing massless bosons, a direct implementation of perturbation theory for the calculation of thermodynamic quantities is problematic. Vanishing Matsubara modes bring, then, infrared divergences that render the naive perturbative series essentially nonconvergent and meaningless [1]. Therefore, the only way to proceed with a sensible perturbative calculation in this realm is to reorganize the diagrammatic series by resumming certain classes of diagrams. In particular, in the early days of the computation of the one-loop effective potential, it was immediately realized that one should resum the so-called ring diagrams even in the limit of weak coupling, and that this recipe provides a thermal mass for the bosonic (thermal) propagator [2]. Resummations of this sort can be performed and improved in different ways. We refer the reader to the reviews of Refs. [3–5] and to Ref. [6] for a longer discussion and a list of more specific references.

In a recent paper [7], we proposed an alternative approach to thermal field theories, which we denoted boundary effective theory (BET). In the present paper, we apply the BET formalism to compute the one-loop effective potential at finite temperature for a system of massless scalar fields with quartic interaction $\lambda\phi^4/4!$. The calculation relies on the solution of the classical equation of motion for the field, and Gaussian fluctuations around it. Our result is nonperturbative and differs from the standard one-loop effective potential [2] for field values larger than $T/\sqrt{\lambda}$.

The boundary effective theory (BET) is a natural way to organize the calculation of the partition function of a quantum system at finite temperature, where one slices the functional space of β-periodic fields into sectors where the boundary value of the field is fixed [7,8]. More precisely, this amounts to writing the partition function Z as

$$Z = \int [D\phi_0(\boldsymbol{x})]\rho_\beta[\phi_0(\boldsymbol{x}), \phi_0(\boldsymbol{x})], \tag{1}$$

where $\phi_0(\boldsymbol{x})$ is the field at the boundaries of the imaginary time interval, and $\rho_\beta[\phi_0(\boldsymbol{x}), \phi_0(\boldsymbol{x})]$ is the functional density matrix diagonal element, given by the functional integration over all the fields $\phi(\tau, \boldsymbol{x})$ that have this particular value at the time boundary,

$$\rho_\beta[\phi_0(\boldsymbol{x}), \phi_0(\boldsymbol{x})] \equiv \int_{\phi(0,\boldsymbol{x})=\phi(\beta,\boldsymbol{x})=\phi_0(\boldsymbol{x})} [D\phi(\tau, \boldsymbol{x})]e^{-S_E[\phi]}, \tag{2}$$

where $S_E[\phi]$ is the Euclidean classical action.

The functional density matrix diagonal element $\rho_\beta[\phi_0(\boldsymbol{x}), \phi_0(\boldsymbol{x})]$ is an expression involving only the static boundary field $\phi_0(\boldsymbol{x})$, which already contains all the temperature dependence. Thus, correlations calculated within this reduced theory encode all information about the thermal distribution of the fields $\phi_0(\boldsymbol{x})$. One can define a dimensionally reduced action $S_d[\phi_0(\boldsymbol{x})]$ through

$$\rho_\beta[\phi_0(\boldsymbol{x}), \phi_0(\boldsymbol{x})] = e^{-S_d[\phi_0(\boldsymbol{x})]}. \tag{3}$$

The reduced theory also contains the relevant infrared physics. Indeed, the double integral structure of the partition function naturally separates the static modes $\phi_0(\boldsymbol{x})$. As a consequence, the reduced theory amounts to a resummation of an infinite class of diagrams of naive perturbation theory. This was verified in Ref. [9], where the effective action obtained in [7] was used to compute the pressure to lowest order in the BET formalism. The resummation of ringlike diagrams emerges directly from the theory, and the corresponding pressure is in good agreement with recent calculations using weak-coupling and screening perturbation theory [10–12].

One should notice that BET is not a particular case of a procedure known in the literature as dimensional reduction (DR) (see Refs. [13–18]). Indeed, DR methods produce effective theories for the zero Matsubara mode and, as such, are high-temperature approximations in character. The procedure that we utilize (BET) yields an *alternative* dimensionally reduced effective theory for the physical field $\phi_0(\boldsymbol{x})$, and it is essentially different from DR.

The aforementioned calculations using the BET formalism are expressed in terms of the classical solutions ϕ_c of the Euler-Lagrange equation. When these classical solutions are computed exactly, i.e., for arbitrary values of the coupling constants, we obtain a nonperturbative result. One diagram in that approach will correspond to a class of perturbative diagrams. We show that BET automatically resums the infinite series of tree diagrams in the strong field regime.

Note that the Euler-Lagrange equations must be solved with nontrivial boundary conditions at the endpoints of the imaginary time interval: $\phi_c(\tau, \boldsymbol{x})$ must be equal to $\phi_0(\boldsymbol{x})$ at the time boundary. To emphasize this functional dependence, we will denote the classical solution by $\phi_c[\phi_0(\boldsymbol{x})]$. In Ref. [7], it is shown that the effective action for the fields $\phi_0(\boldsymbol{x})$ admits a simple expression in terms of the classical solution $\phi_c[\phi_0(\boldsymbol{x})]$. In order to extract the explicit dependence on $\phi_0(\boldsymbol{x})$, one must solve the Euler-Lagrange equation for arbitrary boundary conditions, which is not a feasible task in an interacting theory (but can, in principle, be done numerically).

In this paper, we solve the full classical equation with constant boundary conditions $\phi_0(\boldsymbol{x}) \equiv \phi_0$, allowing for the calculation of the one-loop effective potential of the theory. The complicated nonlinear dependence of ϕ_c on the boundary field ϕ_0 leads to a nonperturbative result that should be a good approximation even at large ϕ_0.

Finally, it should be clear that the use of the word "boundary" has no relation to other uses, often in a topological sense, such as in holographic gauge-gravity duality, etc.

The structure of the paper is as follows. In Sec. II, we revisit the usual prescription to obtain the effective action and the effective potential in the functional integral formalism; using the results of Ref. [6], we write the effective potential in terms of the solution of a certain differential equation. In Sec. III, we apply the method to the massless $\lambda\phi^4/4!$ theory and discuss the results for the effective potential. Finally, in Sec. IV, we present our conclusions.

II. THE ONE-LOOP EFFECTIVE POTENTIAL IN BET

Let us consider the following classical Euclidean action:

$$S_E[\phi] = \int_0^\beta (d^4x)_E \left[\frac{1}{2}(\partial_\mu \phi)^2 + U(\phi)\right], \tag{4}$$

where $(d^4x)_E$ is a shorthand for $d\tau d^3x$. We assume that $U(\phi)$ is some single-well interaction potential. Following the standard procedure for obtaining the effective action, we couple the boundary field to an external current $j(\boldsymbol{x})$, and define the generating functional for the *reduced theory* [19]:

$$Z[j(\boldsymbol{x})] = \int [D\phi_0(\boldsymbol{x})] e^{-S_j[\phi_0(\boldsymbol{x})]}, \tag{5}$$

where

$$S_j[\phi_0(\boldsymbol{x})] \equiv S_d[\phi_0(\boldsymbol{x})] - \beta \int d^3x\, j(\boldsymbol{x}) \phi_0(\boldsymbol{x}) \tag{6}$$

is the action for boundary fields in the presence of $j(\boldsymbol{x})$. We see from Eqs. (2) and (3) that dynamical fields $\phi(\tau, \boldsymbol{x})$ enter into the calculation of $S_d[\phi_0(\boldsymbol{x})]$. In the sequel, we show that a special role is played by dynamical classical solutions.

The free energy functional can be obtained from the generating functional $Z[j(\boldsymbol{x})]$ as

$$F[j(\boldsymbol{x})] = -\frac{1}{\beta} \lim_{V\to\infty} \ln Z[j(\boldsymbol{x})]. \tag{7}$$

By performing a Legendre transform of $F[j(\boldsymbol{x})]$, one formally obtains the effective action

$$\Gamma[\langle\phi_0(\boldsymbol{x})\rangle_j] = F[j(\boldsymbol{x})] + \int d^3x\, j(\boldsymbol{x}) \langle\phi_0(\boldsymbol{x})\rangle_j. \tag{8}$$

Its argument is the expectation value of the field in the presence of the external current. The index j in $\langle\phi_0(\boldsymbol{x})\rangle_j$ is to stress the dependence of the expectation value on the external current. One can think of Γ as *minus* the pressure of the system in response to an external current $j(\boldsymbol{x})$. It can be shown that, at one-loop order, we have $F[j(\boldsymbol{x})] = \Gamma[\langle\phi_0(\boldsymbol{x})\rangle_j]$, and that $\langle\phi_0(\boldsymbol{x})\rangle_j$ is the saddle-point of $S_j[\phi_0(\boldsymbol{x})]$.

In Ref. [7], it was shown that the one-loop effective action for $\phi_0(\boldsymbol{x})$ is given by

$$\beta\Gamma[\phi_0(\boldsymbol{x})] = S_E[\phi_c[\phi_0(\boldsymbol{x})]] + \tfrac{1}{2}\mathrm{Tr}\ln(\Delta_F^{-1} + U''(\phi_c[\phi_0(\boldsymbol{x})])), \tag{9}$$

where Δ_F is the time-ordered thermal propagator and $\phi_c[\phi_0(\boldsymbol{x})](\tau, \boldsymbol{x})$ is the classical solution of the Euler-Lagrange equation for fixed time boundary value $\phi_0(\boldsymbol{x})$, i.e.

$$\begin{aligned} \Box_E \phi_c(\tau, \boldsymbol{x}) + U'(\phi_c(\tau, \boldsymbol{x})) &= 0, \\ \phi_c(0, \boldsymbol{x}) = \phi_c(\beta, \boldsymbol{x}) &= \phi_0(\boldsymbol{x}), \end{aligned} \tag{10}$$

with $\Box_E \equiv -(\partial_\tau^2 + \boldsymbol{\nabla}^2)$ the Euclidean D'Alembertian operator. We see that, when written in terms of $\phi_c[\phi_0(\boldsymbol{x})]$, $\Gamma[\phi_0(\boldsymbol{x})]$ is the same functional as the one-loop effective action at zero temperature. In terms of graphs, Eq. (9) includes the diagrams represented in the Fig. 1. In this figure, the first two terms represent the classical action evaluated at the classical field configuration ϕ_c. The tree structure of ϕ_c when expressed in terms of $\phi_0(\boldsymbol{x})$ is manifest in the Green's formula that relates ϕ_c to its boundary value ϕ_0,

$$\begin{aligned} \phi_c(\tau, \boldsymbol{x}) + \int d\tau' d^3\boldsymbol{y}\, G(\tau, \boldsymbol{x}; \tau', \boldsymbol{y}) U'(\phi_c(\tau', \boldsymbol{y})) \\ = \int d^3\boldsymbol{y}\, \phi_0(\boldsymbol{y}) [\partial_{\tau'} G(\tau, \boldsymbol{x}; \tau', \boldsymbol{y})]_{\tau'=0}^{\tau'=\beta}, \end{aligned} \tag{11}$$

where $G(\tau, \boldsymbol{x}; \tau', \boldsymbol{y})$ is the Green's function of the Euclidean D'Alembertian defined by

$$\begin{aligned} \Box_E^y G(\tau, \boldsymbol{x}; \tau', \boldsymbol{y}) &= \delta(\tau - \tau')\delta(\boldsymbol{x} - \boldsymbol{y}), \\ G(\tau, \boldsymbol{x}; 0, \boldsymbol{y}) = G(\tau, \boldsymbol{x}; \beta, \boldsymbol{y}) &= 0. \end{aligned} \tag{12}$$

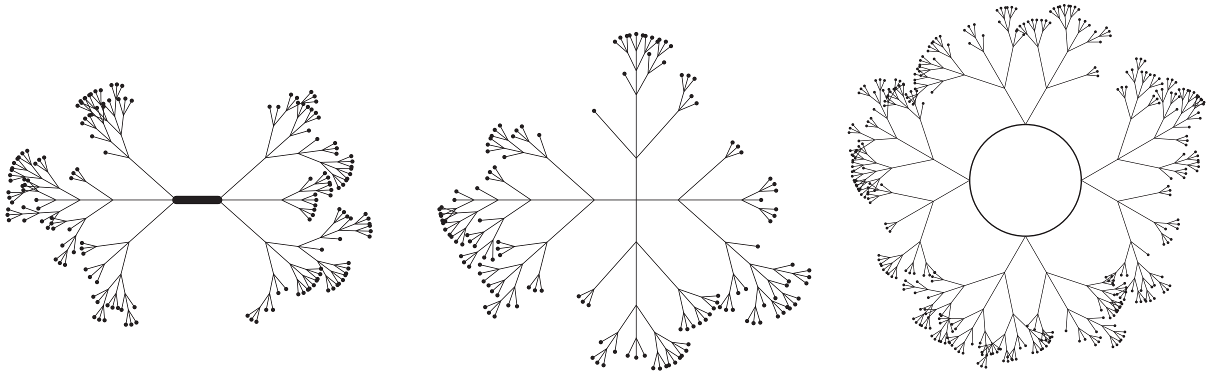

FIG. 1. Diagrammatic representation of the content of Eq. (9). The black dots at the endpoints of the trees represent the boundary field $\phi_0(\boldsymbol{x})$ at which the effective action is evaluated.

The last term in Fig. 1 is the diagrammatic content of the term $\frac{1}{2}\,\mathrm{Tr}\ln(\cdots)$ in Eq. (9). Obviously, the nonlinearities in ϕ_0 are important only at large ϕ_0—when ϕ_0 is small (compared to the temperature), all the trees in Fig. 1 simplify into their lowest order term in ϕ_0 [which is linear in ϕ_0 and amounts to solving Eqs. (10) by neglecting the nonlinear potential $U'(\phi_c)$].

In order to obtain the explicit dependence of $\Gamma[\phi_0(\boldsymbol{x})]$ on $\phi_0(\boldsymbol{x})$, we must solve Eq. (10) for arbitrary boundary conditions, which is in general unfeasible for boundary fields with an arbitrary $\boldsymbol{x}$ dependence. A simpler, yet very useful quantity to compute, is the effective potential, which is essentially the effective action evaluated for uniform boundary field configurations,

$$V_{\mathrm{eff}}(\phi_0) = (\Gamma[\phi_0] - \Gamma[0])/V \quad (\text{for constant } \phi_0(\boldsymbol{x}) \equiv \phi_0), \tag{13}$$

where V is the volume. In this case, the classical solution $\phi_c[\phi_0]$ depends only on τ. Definition (13) is such that $V_{\mathrm{eff}}(0) = 0$. As shown in Ref. [7], the quantity $\Gamma[0]/V$ is the negative of the free pressure, i.e. $(-\pi^2 T^4/90)$.

The calculation of the term $\frac{1}{2}\,\mathrm{Tr}\ln(\cdots)$ in Eq. (9) was done in Ref. [6] as an intermediate step to obtain the pressure in the context of a semiclassical approximation. There, it was shown that this quantity can be expressed, for constant ϕ_0, in terms of solutions of the equation for small field perturbations propagating on top of the classical solution ϕ_c

$$\frac{1}{2}\,\mathrm{Tr}\ln(\Delta_F^{-1} + U''(\phi_c[\phi_0])) = \frac{V}{2}\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\ln[2(\eta(\beta,\boldsymbol{k}^2) - 1)], \tag{14}$$

where $\eta(\tau;\boldsymbol{k})2$ is the solution of

$$[\partial_\tau^2 - k^2 - U\text{''}(\phi_c[\phi_0](\tau))]\eta(\tau,\boldsymbol{k}^2) = 0, \tag{15a}$$

$$\eta(0;\boldsymbol{k}^2) = 1, \qquad \frac{d\eta}{d\tau}(0;\boldsymbol{k}^2) = 0. \tag{15b}$$

Therefore, we obtain the following (nonrenormalized) expression for the effective potential,

$$\beta V V_{\mathrm{eff}}(\phi_0) = S_E[\phi_c[\phi_0]] + \frac{V}{2}\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3} \times \ln[2(\eta(\beta,\boldsymbol{k}^2) - 1)] - \beta\Gamma[0], \tag{16}$$

where the integral over 3-momenta is regularized by the introduction of a cutoff Λ.

In order to renormalize (15), we add the standard one-loop counterterms (C.T.) [obtained by using the same cutoff regularization as the one used in Eq. (16)], and subtract the zero-point energy term $\beta k/2$ from the classical action, obtaining

$$\beta V V_{\mathrm{eff}R}(\phi_0) = S_E[\phi_c[\phi_0]] + \lim_{\Lambda\to\infty}\frac{V}{2}\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3} \times [\ln[2(\eta(\beta,\boldsymbol{k}^2) - 1)] - \beta k] - \mathrm{C.T.} - \beta\Gamma[0], \tag{17}$$

where the counterterms read

$$\mathrm{C.T.} = V\frac{C_1}{2}\int d\tau\,\phi_c^2(\tau) + V\frac{C_2}{4}\int d\tau\,\phi_c^4(\tau), \tag{18}$$

with

$$C_1 \equiv \frac{\lambda}{2}\int^{\Lambda}\frac{d^4k}{(2\pi)^4}\Delta_F^0(k) = \frac{\lambda}{16\pi^2}\Lambda^2 \tag{19}$$

and

$$C_2 \equiv -\frac{\lambda^2}{4}\int^{\Lambda}\frac{d^4k}{(2\pi)^4}\Delta_F^0(k)\Delta_F^0(k+\mu) = -\frac{\lambda^2}{32\pi^2}\left(\ln\frac{\Lambda}{\mu} + \frac{1}{2}\right), \tag{20}$$

where μ is the renormalization scale.

III. RESULTS FOR A MASSLESS THEORY WITH QUARTIC INTERACTION

When $U(\phi) \equiv \lambda\phi^4/4!$, the classical equation of motion is

$$-\partial_\tau^2\phi_c(\tau)+\frac{\lambda}{6}\phi_c^3(\tau)=0, \quad \phi_c(0)=\phi_c(\beta)=\phi_0. \quad (21)$$

The solution of this equation with the required boundary condition is given by

$$\phi_c(\tau)=\sqrt{\frac{6}{\lambda}}\varphi_t\mathrm{nc}(\varphi_t(\tau-\beta/2),1/\sqrt{2}), \quad (22)$$

where nc is one of the 12 Jacobi elliptic functions [20], and φ_t is defined implicitly by the following equation:

$$\phi_0=\sqrt{\frac{6}{\lambda}}\varphi_t\mathrm{nc}(\varphi_t\beta/2,1/\sqrt{2}). \quad (23)$$

Substituting (22) in Eq. (15), we obtain

$$[\partial_\tau^2-\boldsymbol{k}^2-3\varphi_t^2\mathrm{nc}^2(\varphi_t(\tau-\beta/2),1/\sqrt{2})]\eta(\tau,\boldsymbol{k}^2)=0, \quad (24a)$$

$$\eta(0;\boldsymbol{k}^2)=1, \quad \frac{d\eta}{d\tau}(\beta;\boldsymbol{k}^2)=0. \quad (24b)$$

We solve Eq. (24) numerically, and use $\eta(\tau,\boldsymbol{k}^2)$ in Eq. (17) in order to obtain the one-loop effective potential in the BET approach.

The BET effective potential (in units of T^4) is plotted in Fig. 2 as a function of the dimensionless field $\beta\phi_0$ for a coupling constant $\lambda=10$ (the results for $\lambda=1$ are exhibited in Fig. 3). The result is displayed in the form of a band corresponding to a variation of the renormalization scale within the interval $\pi T<\mu<4\pi T$. One can see that the residual sensitivity to the renormalization scale is fairly small, suggesting that higher-order corrections are well under control. However, such a strong claim is based on a first-order calculation and remains risky without evaluating the next-order contribution. In Fig. 3, which shows the results for $\lambda=1$, this band is so narrow that it appears as a line.

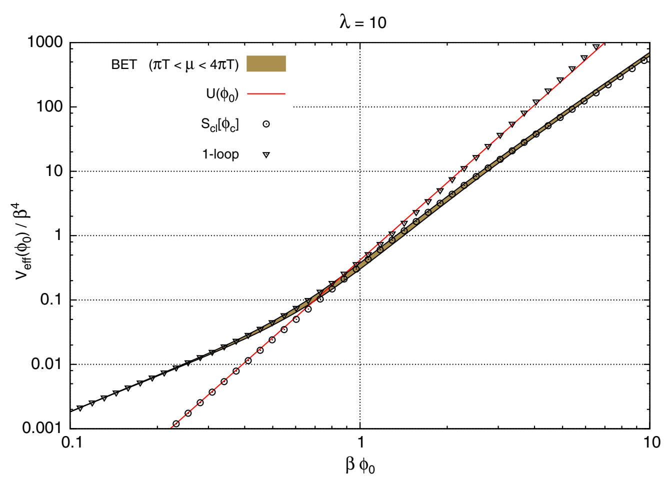

FIG. 2 (color online). Plot of $\beta^4 V_{\mathrm{eff}}(\phi_0)$ for a massless quartic interaction with $\lambda=10$. The shaded band represents the BET result, with a renormalization scale varying in the range $\mu\in[\pi T,4\pi T]$. The thin solid line is the classical potential, i.e. $U(\phi_0)=\lambda\phi_0^4/4!$ in this case. The open circles represent the effective potential obtained from the classical action evaluated at the solution ϕ_c of the Euler-Lagrange equation with boundary value ϕ_0. The open triangles represent the standard one-loop result, evaluated at $\mu=2\pi T$.

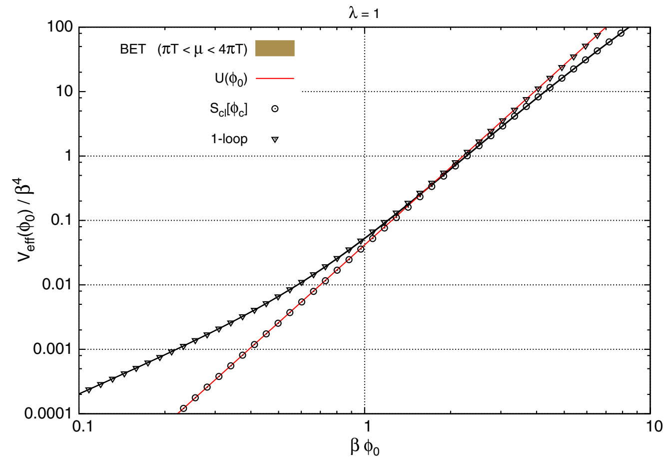

FIG. 3 (color online). Plot of $V_{\mathrm{eff}}(\phi_0)/\beta^4$ for a massless quartic interaction with $\lambda=1$. Definitions are the same as in Fig. 2.

Our result is first compared to the classical potential itself, $U(\phi_0)$. One can see that it differs from the classical potential both at small field, due to the appearance of a quadratic mass term, and at large field due to large nonlinear corrections. At small field, the standard one-loop result and BET are in good agreement, since both incorporate the effect of the thermal mass $m^2=\lambda T^2/24$. However, for larger values of the field, differences appear. In fact, the one-loop result has an asymptotic behavior close to $(\lambda/24)(\beta\phi_0)^4$ (the one-loop correction becomes very small at large field, and one simply recovers the classical potential), while the BET result approaches $\sqrt{\lambda/27}(\beta\phi_0)^3$. One can see that this result is in fact dictated by the behavior of the classical solution $\phi_c[\phi_0]$: when the boundary field $\phi_0(\boldsymbol{x})=\phi_0$ is large, the nonlinear term in the classical Euler-Lagrange equation is very important, and the solution ϕ_c becomes a strongly nonlinear function of the boundary field value ϕ_0. This nonlinearity alters significantly the behavior of the BET effective potential at large ϕ_0. In fact, one can see that in this regime the BET effective potential is well approximated by the classical action evaluated at the solution ϕ_c. This suggests that for large fields, the main effect comes from the infinite sum of the nonlinear tree-level contributions.

To first order in BET, the thermal mass coincides with the Debye mass, as discussed in Ref. [9], reproducing the first-order perturbative result. That means that, for small values of the field, the BET effective potential seems to be as poor as perturbation theory. However, it is a characteristic of semiclassical approximations to be robust in different regimes. In other words, BET is manifestly deficient compared with improvements of perturbation theory for small values of the field, but it can be quite good in a region of larger fields, where perturbation theory does not apply.

IV. CONCLUSIONS

The boundary effective theory (BET) framework provides a way to control the infrared divergences of thermal field theory in a well-defined and relatively simple way [7]. Previously, we had computed the pressure of a massless hot scalar $\lambda\phi^4$ theory [9], obtaining excellent agreement with up-to-date results from weak-coupling and screening perturbation theory [10–12]. In this paper, we have applied this method to the computation of the one-loop effective potential, following our previous work on the semiclassical thermodynamics of scalar fields [6].

The effective potential obtained within the BET formalism perfectly reproduces the standard one-loop result [2] for small fields, as expected, since the BET effective action contains very naturally the effect of the thermal mass. For large fields, BET goes beyond the one-loop approximation by incorporating the nonlinear corrections that become more and more important and are not captured by the standard one-loop calculation. We have also shown that our results are very stable with respect to variations of the renormalization scale, signaling a good behavior of the (semiclassical) series.

A natural, and very useful, extension of this work would be treating the case of thermal symmetry restoration in the case of a double-well classical potential for the scalar field, with its consequences for spontaneous symmetry breaking and the description of phase transitions. However, this is not a straightforward extension, since the case of multiple wells presents nontrivial features related to the appearance of caustics and complex trajectories in the calculation of the semiclassical density matrix (see Refs. [21,22] for a discussion). Nevertheless, we believe that the nonlinear corrections captured by the BET approach can be of great relevance in the description of phase transitions. Results in this direction will be reported in the future [23].

ACKNOWLEDGMENTS

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