We consider superlattices constructed with alternate layers of antiferromagnetic and nonmagnetic materials to study the effect produced by the first nonlinear correction of the susceptibility of the antiferromagnetic material, on the electromagnetic pass bands of these artificial structures. We focus specially on the modifications produced on the pass band edges, as compared to the behavior exhibited under very low magnetic fields. Applications are made for FeF$_2$/ZnF$_2$ superlattices with different thicknesses.

The development of high quality films of antiferromagnetic materials has recently stimulated a large research effort to study nonlinearities in these materials. The main reason for this is that good quality samples exhibiting very narrow resonance widths (few tens of Gauss) in the far infrared region are now available. Therefore it is possible to perform experiments with frequencies very close to the resonance, requiring relatively low incident radiation power to observe nonlinear effects.

Recently the first nonlinear correction to the magnetic susceptibility of antiferromagnets was calculated by Almeida and Mills and their theoretical results for the transmissivity of the electromagnetic radiation through slabs showed that this correction introduces very interesting effects. Kahn et al., using an approach similar to that developed by Delyon et al., extended the calculations of Almeida and Mills and studied the behavior of the transmissivity of superlattices composed by FeF$_2$/ZnF$_2$ as a function of the strength of the incident magnetic field. Their results are very intricate firstly because nonlinear effects are found at low incident power. Secondly there is an incident radiation power threshold, below which the transmissivity depends on the incident field strength in a way very similar to that found for slabs. Above the threshold a chaotic behavior is found. Chen and Mills have studied the nonlinear optical response of dielectric superlattices, the source of the nonlinearity being a power dependence of the dielectric constant in one of the constituent films in the unit cell. Both results are very exciting since they suggest that nonlinear tunable elements may be used, in the future, for applications in the far infrared frequency range. The main aims of the previous works were to examine the origin and nature of the nonlinear response (multistability, for instance) of these systems, for selected values of the incident radiation frequency. The frequency regions were determined from the electromagnetic band structure at very low radiation intensity, where nonlinear effects are absent. The power dependent transmissivity was calculated either in stop gaps or pass bands, as defined by the linear regime (LR).

The objective of this letter is to show modifications introduced in the region of frequency where propagation of electromagnetic radiation through the superlattices is allowed, here called pass bands, when the multilayer system contains in its unit cell an antiferromagnetic material which has a nonlinear susceptibility. We model the superlattice as a stack of alternate layers of dielectric and an (insulating) antiferromagnetic film in the absence of external d.c. magnetic field. In order to have the best insight of the effect of nonlinearity on the pass bands, we choose the thickness of the layers being equal to one half the vacuum wavelength ($\lambda_0$) of a photon of frequency equal to the resonance.
frequency \( \Omega = \gamma ( H^2 + 2H_H H_i )^{1/2} \) of the antiferromagnetic material. We have denoted the anisotropy and exchange fields of the sample by \( H_a \) and \( H_e \) respectively, and \( \gamma \) is the gyromagnetic ratio. Making this choice, we know that the LR pass bands of the superlattice are not very wide in the region where the nonlinear term of the magnetic permeability has significant values. This assures that, within the studied region, there are LR pass bands which will be used for comparison purposes. We do not need a large number of layers to see the stop gap and the pass band structure of an infinite superlattice. In fact it is observed that the asymptotic limit is reached quickly (see for instance figures 2Ca) and 2Cb) in reference (33). The qualitative and quantitative behavior of these regions can be obtained through the analysis of transmissivity of superlattices with a relatively small number of layers.

We consider a plane-polarized electromagnetic wave at perpendicular incidence to the interfaces of the structure, to calculate the power dependent transmissivity. The basic equation is

\[
\frac{d^2 h + k^2 h}{d\Omega} = \frac{1 - \varepsilon_i / \varepsilon_o - 4\pi \chi(\Omega) h}{\varepsilon_o} \quad (1)
\]

where \( h \) denotes the magnetic field of the electromagnetic radiation, \( k \) is the wave vector outside the sample, \( \varepsilon_i \) is the dielectric constant of the ith layer and \( \chi(\Omega) \) is the magnetic susceptibility, which in the nonmagnetic layer is equal to zero and in the antiferromagnetic layer is given by

\[
\chi(\Omega) = \frac{2\gamma^2 M}{(\Omega_o - \Omega^2)} \quad (2)
\]

\[
\Omega_o^2 - \Omega^2 = \varepsilon_i \quad \alpha = \gamma(\Omega_o - \Omega^2) \quad (3)
\]

with \( M \) denoting the saturation magnetization.\(^2\)

Equation (1) is discretized as described in reference (33) to obtain the following recurrence relation between the magnetic field strength at three consecutive points

\[
x_i = x_{i-1} + (F + \alpha x_{i-1} x_{i-2} + 2CF + \alpha x_{i-2}) \quad (i = 0, 1, 2, 3)
\]

where \( F = -2(1 + K^2 \varepsilon_i / \varepsilon_o) \), \( K = k_o (dz) \), with \( dz \) the width of the interval used to discretize (1). \( \alpha \) is the nonlinear coefficient of equation (1) and \( x_n \) (n = 1,2,3) is a dimensionless variable denoting the magnetic field of the radiation in units of the strength of the incident magnetic field multiplied by the transmissivity.

Equation (1) is valid for \( x_1, x_2 \) and \( x_3 \) in the same medium (layer). To proceed from one medium to the next the recurrence relation is obtained from the boundary conditions for the magnetic field which in the discretized form is given by

\[
(x_i - x_{i-1})/\varepsilon_o = (x_i - x_{i-2})/\varepsilon_o \quad (4)
\]

where \( \varepsilon_i \) and \( \varepsilon_o \) denote the dielectric constants of the consecutive layers. The calculation is initialized at points just outside the superlattice at the end opposite to incidence where \( x_1 \) and \( x_2 \) are equal to one as a result of the units chosen.

In order to study the modifications of the pass bands edges, we calculate the power dependent transmissivity in a given frequency region as a function of the incident radiation intensity. The pass bands are characterized by transmissivities above the noise level observed in the LR stop gaps. A large sampling of the frequency/intensity plane was made and the results for the LR were reproduced with great accuracy. We find that there are regions (in the frequency/intensity plane) where the power dependent transmissivity has a stop gap behavior and regions where it has a pass band behavior. The most interesting results are shown in Fig.1. Here we select a particular LR pass band frequency region for the FeF/ZNFe superlattice containing 16 layers (the values for \( M, H_a \) and \( H_e \) for FeF are 0.86 kG, 200 and 340 KOe, respectively). We observe that for very low incident radiation intensity the pass band has the same width of the linear case and as the magnetic field intensity is increased the band edges raise to higher frequencies. The character of the different regions shown in Fig.1 is confirmed by the behavior of the power dependent transmissivity along a constant frequency line. For comparison purposes in Fig.2Ca) we show the power dependent transmissivity for a frequency inside a pass band (\( \Omega_o - \Omega/\gamma = 270 \) Gauss). We observe that for the same multistability shown previously in antiferromagnetic slabs\(^2\) in Fig.2Cb) and Fig.2Cc) we show the transmissivity as a function of the magnetic field strength for two other selected frequencies. In Fig.2Cb) (solid curve), where the pass band edge is approached from the left, for a value of \( \Omega_o - \Omega/\gamma = 265 \) Gauss, we...
observe that the transmissivity has two different behaviors. For magnetic field strength (MFS) lower than 0.35 Gauss the transmissivity is within the noise level characteristic of stop gaps and in the second region, above this value of the MFS, the pass band nonlinear character of T is clearly shown. This corresponds exactly to what one expects from Fig. 1. In Fig. 2(c), for \( (\Omega - \omega)/\gamma = 360 \) Gauss, we observe just the opposite behavior. For MFS lower than 1.73 Gauss a typical pass band transmissivity is found, whereas above this value the transmissivity goes to zero. In Fig. 2(b) (dashed curve) a different frequency is chosen; \( (\Omega - \omega)/\gamma = 295 \) Gauss. We find that, for this frequency, the transmissivity exhibits the same behavior shown for \( (\Omega - \omega)/\gamma = 285 \) Gauss. For MFS lower than 1.40 Gauss the transmissivity is very low and characteristic of stop gaps, whereas for MFS above 1.40 Gauss we find a pattern characteristic of pass bands, as expected from Fig. 1. It is worth mentioning that for this frequency value, a much bigger MFS is required for the pass band behavior to be attained.

We have also considered a more realistic \( \text{FeF}_2/\text{ZnF}_2 \) superlattice structure consisting of eighty layers, each of which with a thickness ten times smaller than the previous. We obtained the same pattern as in Fig. 1, with pass bands and stop gaps depending on the incident radiation intensity in a similar way. In Fig. 3 we show the map corresponding to a selected frequency region, near the bottom of a LR pass band. We can see clearly how the pass band bottom is altered by the magnetic

**Figure 1** - Pass bands (shaded area) of the \( \text{FeF}_2/\text{ZnF}_2 \) superlattice with 16 layers \( \lambda/2 \) thick (SL1), as a function of incident MFS.

**Figure 2** - Power dependent transmissivity of the SL1 as a function of the incident MFS. (a) Inside the pass band (PB) with \( (\Omega - \omega)/\gamma = 270 \) Gauss (b) Approaching the PB from the left with \( (\Omega - \omega)/\gamma = 285 \) Gauss (solid curve) and \( (\Omega - \omega)/\gamma = 295 \) Gauss (dashed curve). (c) Going out of the PB with \( (\Omega - \omega)/\gamma = 360 \) Gauss.

**Figure 3** - Pass bands of the \( \text{FeF}_2/\text{ZnF}_2 \) superlattice with 80 layers \( \lambda/20 \) thick (SL2), as a function of the incident MFS (shaded area). Power dependent transmissivity of the SL2 as a function of the incident MFS for a frequency of \( (\Omega - \omega)/\gamma = 940 \) Gauss (approaching the PB from the left).
field. In Fig. 3 we also show the power dependent transmissivity (left hand axis) for a single frequency \((\Omega - \Omega_0)/\gamma = 0.6\) Gauss and the power dependent pass band edge is clearly seen for a MFS of 5.0 Gauss.

It is clear that the nonlinear response of the antiferromagnet constituent of the superlattice produces a field dependent magnetic susceptibility. As a result one has a position dependent index of refraction and therefore the band structure depends on the incident radiation intensity. The dielectric constants and thicknesses of each constituent of the superlattice contribute to alter the band structure as well but, for a fixed set of geometric and linear parameters, all modifications come from the nonlinear effects. Our numeric procedure is a very powerful and versatile method capable of producing the radiation pass band structure and particularly its dependence on the incident MFS.

We have shown numerical results for FeF\(_2\)/ZnF\(_2\) superlattices. However, other antiferromagnetic materials such as MnF\(_2\) and CoF\(_2\) may be considered and the same qualitative behavior is expected. Thin films of these materials are now available and we hope that experiments can soon be made on new superlattices, bringing more information about the fascinating characteristics of these artificial structures.

REFERENCES