Effects of graphene on light transmission spectra in Dodecanacci photonic quasicrystals


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A B S T R A C T

We make a theoretical study of light waves, for both TE and TM polarization with oblique incidence. The light waves propagate on SiO2/TiO2 multilayers, organized by Dodecanacci sequence, where we consider a graphene monolayer at the interfaces between distinct layers. We also calculate numerically the optical transmissivity spectrum, by using the transfer matrix method, for the cases with a graphene sheet and without a graphene sheet at the interfaces. Our results indicate that the whole optical spectrum becomes affected by the presence of graphene in the interfaces. Among these effects, we can highlight the shift of bandgaps to high-frequency regions, the emergence of a graphene induced bandgap at low-frequency regions, and the decreasing of transmittance in the whole frequency range considered here. The self-similarity, which is a property that indicates a fractal spectrum, keeps essentially unchanged when we consider graphene sheets between the layers. Our studies can be applied to electromagnetic filters, where an input gate voltage, applied on the graphene sheets, can control the bandgaps which arise on these structures.

1. Introduction

After the studies of E. Yablonovitch in 1987 [1], the possibility of controlling the propagation of electromagnetic waves has been receiving much attention. In his studies, Yablonovitch considered a semiconductor with intrinsic properties, among which stand out its ability to guide and confine light. This new artificial material became known in the literature as a Photonic Crystal (PC). The PCs are structures that have a very definite periodicity and can be artificially constructed using one or more dielectrics of different permittivity ε in the structure of the multilayer. The property of allowing and prohibiting the propagation of light into a multilayer gives rise to the so-called photonic bandgaps (PBGs). These PBGs that arise in these structures are a direct consequence of the Bragg scattering [1].

The crystals possess a fundamental property that characterizes them: translational symmetry. However, recently a third class of materials was discovered and characterized, in addition to the already known amorphous and crystalline ones. The PCs can be treated as photonic quasi-crystals (PQCs) if the spatial distribution of their unit cells or their refraction indices is organized in a quasi-periodic manner [2]. Usually, this organization is made under a mathematical rule of substitution. They are structures known for the absence of translational symmetry, or for possessing translational symmetry of prohibited order: 5, 8, 10 and 12 [3], which differs from conventional crystals of order: 1, 2, 3, 4 and 6. The discovery of quasicrystals (QCs) is attributed to Dan Shechtman et al., in 1984, and for such studies, he won the Nobel Prize in chemistry in 2011 [4]. Shechtman and colleagues presented a metallic alloy that exhibited a diffraction pattern different from the usual. Although the diffraction results display a monocrystalline structure, it exhibited a non-translational periodicity or translational periodicity of prohibited order. However, it was observed that these structures possess long-range order and exhibit a similarity behavior. Also, Levine and Steinhardt were awarded the Buckey Physics award for their theoretical analysis of QCs [5].

One of the major motivations in the study of QCs is the fact that they present a fragmented spectrum, which exhibits a pattern of self-similarity [6]. Self-similarity is a very important property in the study of fractals. In general, aperiodic structures lack symmetry; however, for
the case of QCs, X-ray diffraction exhibits a fairly crisp pattern, just as those of regular periodic lattices [7]. In this work, we will study the quasicrystals constructed according to the Dodecanacci substitution rule, to form the Dodecanacci photonic quasicrystals (DPQCs). Such structures have a translational symmetry of prohibited order of degree 12 and can be constructed according to the substitution matrix found in Ref. [8].

On the other hand, the search for new optical devices with different properties had a significant advance with the development of studies on graphene. Graphene is a two-dimensional structure formed by atoms hybridized in sp² that form a hexagonal network known as honeycomb. The properties of these compounds formed by carbon have been widely reported [9]. Graphene had its first appearance in science through the first theoretical studies carried out in 1947 by Wallace [10]. However, it was only in 2004 that graphene was obtained experimentally through a group of scientists led by Geim and Novoselov [11]. From a technological point of view, several impressive properties are attributed to graphene. It is well known, for example, that graphene has excellent transport properties, a characteristic that makes it an excellent conductor. From a mechanical point of view, among the known materials, graphene is one of the finest and most resistant materials. Moreover, concerning the optical properties, it is known that graphene absorbs little light, making them practically transparent conductors (for a review see Ref. [12]).

In this work, it will be considered that, between the interfaces of the layers, we have a graphene sheet, characterized by a conductivity σγ. Thus, we have a quasicrystal of Dodecanacci graphene (QCDG). We will study the effect of placing this material on those multilayer systems. Specifically, we will study the photonic quasicrystals of graphene constructed using the juxtaposition of blocks according to the Dodecanacci rule. Also, we will use the transfer matrix technique to study the transmission spectrum of these structures.

2. Theoretical model: Transfer-matrix method and Dodecanacci sequence

In the present work, a theoretical model, based on the transfer-matrix treatment (for a review see Refs. [2,13]) is applied to calculate the optical transmission (or reflection) spectra of a multilayer system presented in Fig. 1. Therefore, consider a s-polarized (or p-polarized) light of frequency ω, obliquely incident from a transparent medium C at an arbitrary angle θc with respect to the normal direction of the layered system (see Fig. 1). The multilayered system is formed from an array of slabs of different materials (A and B), which corresponds to two different non-magnetic materials namely, SiO₂ (silicon dioxide) and TiO₂ (titanium dioxide). The non-dispersive dielectric constants of the media A, B, and C are, respectively, εA, εB, and εC. The reflectance and the transmittance coefficients are given, respectively, by

\[
\begin{align*}
R &= \frac{M_2}{M_1}, \\
T &= \frac{1}{M_1},
\end{align*}
\]

where \(M_{ij}\) are the elements of the optical transfer-matrix \(M\), which will be straightforwardly deduced for our case in subsection 2.3.

This matrix links the coefficients of the electromagnetic fields in the region \(z < 0\) to the coefficients of the electromagnetic fields in the region \(z > L\), \(L\) being the size of the quasi-periodic structure that we consider here.

Also, at each interface between different materials, it is considered that there is a graphene sheet (with the same Fermi energy) whose thickness is negligible compared to \(d_A\) and \(d_B\) (the thickness of the layers \(A\) and \(B\), respectively). We have modeled the graphene sheet by the frequency-dependent optical conductivity, that is given by \(\sigma = \omega \sigma_{\text{para}} \sigma_{\text{inter}}\), where

\[
\begin{align*}
\sigma_{\text{para}} &= \frac{\varepsilon^2}{4\hbar} i \frac{16k_B T}{\hbar \omega} \ln \frac{\hbar \omega}{\mu_I k_B T^2}, \\
\sigma_{\text{inter}} &= \frac{\varepsilon^2}{4\hbar} \frac{1}{\tan \frac{1}{2}} \frac{\hbar \omega}{2\mu_I k_B T^2} \ln \frac{\hbar \omega}{\mu_I k_B T^2},
\end{align*}
\]

which includes both intraband (\(\sigma_{\text{para}}\)) and interband (\(\sigma_{\text{inter}}\)) terms. Here \(\varepsilon\) is the electronic charge, \(T\) is the absolute temperature, \(k_B\) is the Boltzmann constant, and \(\mu_I\) is the chemical potential. The interband contribution plays an important role around the absorption limit, \(\hbar \omega = 2\mu_I\), while the intraband contribution is important at low frequencies compared with \(\mu_I / \hbar\), giving a competing effect. In this paper we will consider high frequencies, far from the absorption limit (mid-infrared). Therefore, in that case, the interband contribution will be more important than intraband contribution. However, for completeness, we prefer to keep the above expression as a general case.

2.1. TE modes

In order to obtain the transfer matrix for the photonic multilayer we consider the electromagnetic wave for transverse electric field (TE) modes (or s-polarization) within medium A (or B) of the \(l\)-th layer, can be given by

\[
E_{\text{TE}}^{\text{l}} \left[ A_j^{l(\text{TE})} \exp \left( i k_{j(\text{TE})} z \right) A_j^{l(\text{TE})} \exp \left( i k_{j(\text{TE})} z \right) \right] \exp \left( ik_{\text{x(TE)}} \right),
\]

where \(A_j^{l(\text{TE})}\) and \(A_j^{l(\text{TE})}\) (\(j = C, A\) or \(B\)) are the amplitudes for the forward- and backward-travelling waves, respectively. Also, the superscript index \(l\) label the electrical fields in each medium (with \(l = 0\) and \(l = n\) being the external medium \(\varepsilon_C\)), in such a way that \(z_0 = z_1 = z_2 = 0\) is equal to \(d_A + d_B\), if \(l\) is odd or even, respectively. Using \(H \equiv \varepsilon_0 c^2 / \omega \nabla \times \varepsilon C \times (\hat{E})\), the magnetic field in the \(l\)-th layer is written as

\[
H_{\text{TE}}^{l(\text{TE})} \left[ \frac{\varepsilon_c}{\mu_{\text{r(TE)}}} \left[ A_j^{l(\text{TE})} \exp \left( i k_{j(\text{TE})} z \right) A_j^{l(\text{TE})} \exp \left( i k_{j(\text{TE})} z \right) \right] \exp \left( ik_{\text{x(TE)}} \right),
\]

where \(\mu_0\) is the vacuum permeability.

Applying Maxwell’s boundary conditions, namely, continuity of the tangential component of the electric field \(E_{\parallel}^{\text{l(TE)}}\) and the discontinuity of

Fig. 1. (Color online) The geometrical schematic representation of the multilayered photonic structure considered in this work. This structure corresponds to the second generation of Dodecanacci sequence, where the unit cell is \(S_2\). The blue layers A (SiO₂) and the yellow layers B (TiO₂) have thicknesses \(d_A\) and \(d_B\), respectively, forming the building blocks of the whole quasi-periodic structure. Also, we consider graphene only at \(A,B\) interfaces. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)
the magnetic field (that is equal to the current density $\vec{J}$) at the interface, $z=0$ (see Fig. 1), we can relate the electromagnetic fields between the first layer with their neighbor (medium C in this case). Rewriting the amplitudes in terms of column vectors, we obtain the following matrix equation for a $C/A$ interface,

$$
\begin{pmatrix}
A_{1A} \\
A_{1C}
\end{pmatrix} = M_{CA}
\begin{pmatrix}
A_{1A} \\
A_{1C}
\end{pmatrix}
$$

(6)

where the matrix $M_{CA}$ represents the waves crossing the interface C/A and the $M_{C}$ is the propagation matrix in the medium C (for more details, see Ref. [14]). Applying Maxwell’s boundary condition for all interfaces, we have the transfer matrix for the whole multilayered system, $M = M_{C}M_{CA}M_{MA}M_{A}...$. We can generalize this for a $\alpha/\beta$ interface, and we have the crossing matrix given by

$$
M_{\alpha/\beta} = \begin{pmatrix}
\frac{n_{\beta}}{n_{\alpha}} & 1 \\
1 & \frac{n_{\alpha}}{n_{\beta}}
\end{pmatrix}
\begin{pmatrix}
1/k_{\alpha} & k_{\beta} \\
k_{\beta} & 1/k_{\alpha}
\end{pmatrix}
\begin{pmatrix}
1 & \frac{\mu_{\beta}}{\mu_{\alpha}} & 1 \\
\frac{\mu_{\alpha}}{\mu_{\beta}} & 1 & \frac{\sigma_{\beta}}{\sigma_{\alpha}} \\
1 & \frac{\sigma_{\alpha}}{\sigma_{\beta}} & 1
\end{pmatrix}
$$

(7)

where

$$
k_{\alpha} = \frac{n_{\alpha}}{c} \sin \theta,
$$

(8)

and

$$
k_{\beta} = \frac{n_{\beta}}{c} \sin \theta,
$$

(9)

where $c$ is the speed of light in vacuum. Here we have used Snell’s law and considered the in-plane wavevector for all layers to be equal, i.e., $k_{\alpha} = k_{\beta} \sin \theta_{\alpha} = k_{\beta} \sin \theta_{\beta} = k_{\beta} \sin \theta$, with $k_{\beta} = n_{\beta}/c$. Also, the propagation matrix can be generalized for any layer $\gamma$ (i.e., $A$ or $B$), and it is given by Ref. [14],

$$
M_{\gamma} = \begin{pmatrix}
\exp i k_{\gamma} d_{\gamma} & 0 \\
0 & \exp i k_{\gamma} d_{\gamma}
\end{pmatrix}
$$

(10)

where $d_{\gamma}$ is the thickness of the respective material.

2.2. TM modes

For the case of transverse magnetic (TM) waves ($p$-polarization) we assume that the $x$-component of the electric field for the $l$-th slab is given by

$$
E_{\|}^{l} = \left[ A_{l} \exp i k_{\|} z + \bar{A}_{l} \exp i k_{\|} z \right] \exp i k_{\perp} z \ i\omega t,
$$

(11)

Using $\vec{H} = -i\omega \epsilon_{0} \vec{E}$, the magnetic field in medium $A$ (or B) in the $l$-th layer is obtained as

$$
H_{\|}^{l} = \frac{\epsilon_{0} \epsilon_{\|}}{k_{\|}} \left[ A_{l} \exp i k_{\|} z + \bar{A}_{l} \exp i k_{\|} z \right] \exp i k_{\perp} z \ i\omega t,
$$

(12)

where, without loss of generality, we consider the magnetic permeability of each medium equal to unity, i.e., $\mu = 1$. Also, here we have used $\vec{\nabla} \cdot \vec{D} = 0$, to relate the amplitudes of the electric and magnetic fields.

Following the same steps as for the s-polarization, we can apply the standard Maxwell’s boundary conditions (continuity of the tangential components of $\vec{E}$ and discontinuity of $\vec{H}$) at the interfaces $z=0$ and $z=d_{4}$ (see Fig. 1), so we can obtain the crossing matrix for TM waves given by Ref. [15],

$$
M_{\alpha/\beta} = \begin{pmatrix}
1 & \frac{1}{\mu_{\alpha}} & 1 \\
\frac{1}{\mu_{\beta}} & 1 & \frac{1}{\mu_{\alpha}}
\end{pmatrix}
\begin{pmatrix}
1 & \frac{\epsilon_{\|}}{\epsilon_{\perp}} & 1 \\
\frac{\epsilon_{\perp}}{\epsilon_{\|}} & 1 & \frac{\sigma_{\|}}{\sigma_{\perp}} \\
1 & \frac{\sigma_{\perp}}{\sigma_{\|}} & 1
\end{pmatrix}
$$

(13)

where $\lambda$, $k_{\|}/\sigma_{\perp}$, $\alpha$, $\beta$.

2.3. Dodecanacci photonic quasi crystals

A Dodecanacci multilayer photonic structure is defined by the juxtaposition of two building blocks $A$ ($\text{SiO}_{2}$) and $B$ ($\text{TiO}_{2}$), where the $n$th-generation of the multilayer $S_{n}$ is generated iteratively by the rule $S_{n}$ $A_{n} S_{n} \ 2S_{n} \ 2S_{n}$, for $n = 3$, with $S_{1}$ AAB and $S_{2}$ $A_{2} S_{2} \ 2S_{2}$ [8]. In Fig. 1, we present the spatial distribution of the building blocks $A$ and $B$ according to the second generation of a Dodecanacci sequence. The number of the building blocks increases according to Dodecanacci number $D_{n}$ $4D_{1} 1 D_{2}$ (with $D_{1} = 3$, $D_{2} = 11$ and $n = 3$). Also, the characteristic value of a Dodecanacci sequence, defined as the ratio $D_{n}$ to $D_{n-1}$ in the limit $n \to \infty$, is $\tau$ = lim$_{n \to \infty}$ $D_{n}/D_{n-1}$ $2 \sqrt{3}$, which is the positive solution of the quadratic equation $x^{2} - 4x + 1 = 0$. An alternative way to obtain this sequence is by using the substitution rule $A \to \text{AAB}$ and $B \to \text{AAB}$.

Now, by considering the iterative rule for the Dodecanacci photonic quasicrystals, it is easy, after some algebra, to determine the following general formula for the transfer-matrices in the Dodecanacci quasiperiodic system,

$$
M_{n} = M_{CA} \ M_{BC}, \quad \text{for} \quad n = 3
$$

(14)

where

$$
T_{\alpha} = T_{\alpha} T_{\beta}(A_{\beta} A_{\beta} 2T_{\alpha} 1) \quad \text{and} \quad T_{\alpha} = M_{\alpha} T_{\alpha} m = 1
$$

(15)

whose initial conditions are $T_{\alpha}$ $M_{\alpha} A_{\beta} A_{\beta} 2T_{\alpha} 1$ and $T_{\alpha} = M_{\alpha} T_{\alpha} m = 1$, with (14) and (15) we can calculate numerically the transmission spectra as a function of the angular frequency $\omega$, or the wavelength $\lambda = 2\pi c/\omega$, for a given incident angle in Eq. (9). The next section is reserved to applications of this numerical calculation.

3. Numerical results

In this section, we present the numerical results that we have obtained concerning the light transmission spectra of Dodecanacci quasiperiodic photonic multilayered structures (see Fig. 1). In this work we have considered medium $A$ as silicon dioxide ($\text{SiO}_{2}$), and $B$ as titanium dioxide ($\text{TiO}_{2}$), with their respective nondispersive refractive indexes $n_{A} = 1.45$ and $n_{B} = 2.30$ [2,16]. Furthermore, we take into account that the individual layers satisfy the quarter-wavelength condition, i.e., $n_{\alpha} d_{\alpha} = n_{\beta} d_{\beta} = \lambda_{0}/4$, and we do expect the quasiperiodic effects to be more apparent. Consequently, for a central wavelength $\lambda_{0} = 60 \mu m$, the thickness of the dielectric slabs are $d_{\alpha} = 60/4n_{A} = 10.34 \mu m$ and $d_{\beta} = 60/4n_{B} = 6.52 \mu m$. Besides, the semi-infinite medium $C$ is considered as vacuum ($n_{C} = 1$), and we have defined a reduced frequency $\Omega = \omega/\omega_{0}$ $= \lambda_{0}/\lambda$, with $\omega_{0} = 2\pi c/\lambda_{0}$.

The light transmission spectra, as a function of the reduced frequency $\omega/\omega_{0}$, for a one-dimensional Dodecanacci photonic quasicrystal are presented in Fig. 2, for a normally incident wave, i.e., $\theta_{C} = 0$. The labels (a)–(d) correspond to Dodecanacci 1D-PQC without graphene at their interfaces (solid blue lines), while labels (e)–(h), to the structure with graphene at every $A/B$ and $B/A$ interfaces, and considering the chemical potential $\mu_{\|} = 0.2 eV$ (solid red lines). We firstly discuss the case without graphene. The transmission for the fifth generation, where $D_{5} = 571$ layers, is presented in Fig. 2a. As it is expected for normal incidence, the transmission spectrum posses a mirror symmetry, around the mid-gap reduced frequency $\omega/\omega_{0} = 1$, because of the quarter-wavelength condition for a periodic multilayer. In all sequences investigated here, we observe that the photonic quasicrystal is quite transparent in the mid-gap reduced frequency, i.e., $T \approx 1$ for $\omega/\omega_{0} = 1$. As an opposition to this behavior, in previous works we have shown that light is totally reflected in the mid-gap frequency on Octonacci photonic quasicrystals [17], while the transmission oscillates between 0 and 1, depending on...
the generation number, on Fibonacci ones [2]. Furthermore, as generally occurs for any wave phenomena in quasiperiodic heterostructures, the transmission spectrum presents a scaling property with respect to the generation number of Dodecanacci sequence, within a symmetrical interval around the mid-gap reduced frequency \( \omega/\omega_0 \). In order to better explain this point, we present in Fig. 2b the transmission spectra shown in Fig. 2a, but for the reduced range of frequency 0.99 \( \omega/\omega_0 \) to 1.01. This spectrum is similar to the one representing the sixth generation of the quasiperiodic Dodecanacci sequence (where \( D_6 \) 2131 layers), displayed in Fig. 2c for the range of frequency reduced by a scale factor equal to 10. In Fig. 2d, we present the transmission spectra for seventh generation (where \( D_7 \) 7953 layers) for 0.9997 \( \omega/\omega_0 \) to 1.0003, which corresponds to a scale factor related to Fig. 2c of approximately 3.33, and it is similar to Fig. 2b and c, and we can observe that such spectra repeat every generation for a Dodecanacci one-dimensional photonic quasicrystal as it occurs with Octonacci sequence [17]. This scaling property corresponds to a self-similar behavior of the spectrum, which is a qualitative evidence of a fractal spectrum.

The light transmission spectra, as a function of reduced frequency \( \omega/\omega_0 \) and with \( \theta_0 = 0^\circ \), for the case in that graphene is placed at every \( AB \) and \( BA \) interfaces are also shown in Fig. 2, for the same generations of Dodecanacci sequence, i.e., (e) fifth generation, (f) zoom of fifth generation for the reduced range of frequency 0.99 \( \omega/\omega_0 \) to 1.05, (g) and (h) correspond to sixth and seventh generations for 1 \( \omega/\omega_0 \) to 1.04 and 1.005 \( \omega/\omega_0 \) to 1.035, respectively. The presence of graphene at the interfaces between different media causes the loss of the mirror symmetry on the transmission spectra around the mid-gap reduced frequency \( \omega/\omega_0 \) to 1, although the quarter-wavelength condition was considered, as it was deeply discussed on previous works [14,18,19]. The central frequency has now a new value of reduced frequency around 1.02. As it occurred in other cases, we see from Fig. 2e that graphene induces the emergence of a non-Bragg’s gap called graphene induced bandgap (GIBG) at low-frequency region, namely, \( \omega/\omega_0 < 0.287 \), which corresponds to \( \omega < 9 \) THz. Note that transmission scales on (g) and (h) are from 0 to 0.5 and from 0 to 0.05, respectively, as a consequence of the increase in the number of generations, which also corresponds to an increase in the amount of graphene in the structure. This result implies that a rapid decrease in light transmission is observed, as a result of an enormous increase in absorption, regarding the high absorption coefficient due to graphene [14].

Now, we treat the cases of obliquely incident light waves. In Fig. 3, we plot the transmission coefficient \( T \) as a function of the reduced frequency \( \Omega \) and the incident angle \( \theta_0 \) (in degrees) for transversal electric or TE waves (left) and transversal magnetic or TM waves (right): (a) third, (b) fourth, (c) fifth and (d) sixth generations of Dodecanacci sequence. These plots correspond to the case without graphene. In all these figures, the white (black) color means a transmission coefficient equal to 1 (0). Unlikely of the Fibonacci and Octonacci sequences, the omnidirectional photonic bandgap gap (OPBG), which is a gap region that appears in both TE and TM waves for all incidence angles, was not observed for Dodecanacci one. In comparison to other substitutional sequences, we conclude that this omnidirectional gap is a consequence of the low disorder produced by the substitution rule. Considering only these three sequences of the “Nacci” family: Fibonacci, Octonacci, and Dodecanacci, this last one has the highest degree of disorder, and as a consequence, this causes the absence of the omnidirectional photonic gap.

In Fig. 4, we display the same plots as in Fig. 3, but now including graphene at every \( AB \) and \( BA \) interfaces, and with the chemical potential \( \mu \) adjusted to 0.2 eV. The only well-defined OPBGs correspond to the GIBGs, enhanced by the green rectangles, for a reduced frequency \( \omega/\omega_0 < 0.287 \), which are almost independent of both the incident angle and the generation number. By comparing the color scale of the transmittance \( T \) on Fig. 4a–d, we can also see in a more clear way how quickly the transmission coefficient decreases as the generation number increases. This is connected to the fact that the number of building blocks on Dodecanacci sequence grows very fast.

In Figs. 5–7, we plot the transmission coefficient for TE (left) and TM (right) waves versus the reduced frequency \( \Omega \) and the chemical potential \( \mu \) for a given incident angle, namely, \( \theta = 25^\circ, \theta = 50^\circ \) and \( \theta = 75^\circ \), respectively. For all these Figures, in (a) we have 3rd, in (b) we have 4th, in (c), we have 5th and, finally, in (d) we have 6th generations of Dodecanacci sequence. For \( \theta = 25^\circ \), Fig. 5, we do not observe an appreciable difference between TE and TM transmission spectra because the incidence angle is still close to the normal incidence case. As the incidence angle increases, for instance, taking \( \theta = 50^\circ \), displayed in Fig. 6, the expected difference between the transmission coefficient and bandgaps position for TE and TM waves becomes more pronounced, with a higher transmission for TM than for TE waves, but the photonic
bandgaps are narrower for TM than for TE polarization. For $\theta = 75^\circ$, the light transmission spectra for TE and TM waves, presented in Fig. 7, are strongly influenced by graphene: firstly, we observe that the transmission coefficient is lower for TE waves than for TM ones; secondly, we observe that there are some frequency regions which correspond to bandgaps for TE polarizations and allowed bands for TM waves, for example, at the spectra for reduced frequencies around 0.75 and 1.5. This characteristic is advantageous to design light polarizing filters based on photonic crystals. Also, in Figs. 5–7, we note a monotonic increase in the width of the lowest gap induced by graphene and a high frequencies shift of other gaps as the chemical potential increases. This fact makes both the width and position of the gaps controllable and adjustable, by an external gate potential, which constitutes a practical way to get a high control over the design of photonic crystals with specific pass- and stop-bands structure.

4. Conclusions

The main objective of this work was the theoretical study of photonic transmission spectra using the transfer matrix technique, for a one-
dimensional Dodecanacci quasi-crystal with graphene. Initially, for the case of structures without graphene, we obtained that for the normal incidence of light ($\theta = 0^\circ$), the transmission spectrum has a mirrored symmetry around the reduced frequency $\omega / \omega_0 = 1$. We could observe a scaling behavior with respect to the number of generations $n$ of the Dodecanacci sequence, for different reduced frequency values. This scaling property corresponds to a self-similar behavior of the spectrum, which is a qualitative evidence of a fractal spectrum, a result that is analogous to the one previously found for the Octonacci sequence [18].

When we consider graphene between the $A/B$ and $B/A$ interfaces, for normal incidence ($\theta = 0^\circ$), we note the breakdown of the mirror-like symmetry around the reduced frequency $\omega / \omega_0 = 1$. We noticed that the central gap was shifted slightly to the right, with a new value of the reduced frequency at 1.02. In low-frequency regions, the graphene sheet induces a bandgap of “non-Bragg” nature, known as graphene-induced bandgap (GIBG). Another effect of graphene on the multilayer structure is the reduction of the transmittance spectrum. This decrease becomes more pronounced with the increase in the number of generations $n$, which directly influences the number of graphene sheets in the quasi-crystal, and consequently, there is a huge increase in the absorption of light [14]. In the case of the oblique incidence of light, without graphene, we noticed the absence of an omnidirectional bandgap. We conclude that this omnidirectional bandgap occurs for low degrees of disorder in relation to the substitution rule, which does not occur in the...
sequence of Dodecanacci. When we consider graphene in the multilayer structure, we note that the only omnidirectional photonic bandgap (OGBP) that exists corresponds to the GIBG. Afterward, we calculated the transmission coefficient for TE and TM waves, with respect to the reduced frequency $\Omega$ and chemical potential $\mu_c$, for different angles of incidence, and we studied the influence of the chemical potential in the spectrum. For angles near normal incidence, $\theta \approx 25^\circ$, in our case, we did not observe remarkable differences between TE and TM waves, but for higher angles, $\theta > 50^\circ$ and $\theta > 75^\circ$, for instance, we noticed an increase in the width of the GIBG and a shift to higher frequencies in the remaining gaps. This control of the width and position of the gaps, in the multilayer structure via an external gate, becomes a very effective tool for the control and guide of light in the quasi-crystalline structure.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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