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Band gaps in the terahertz frequency range in quasiperiodic one-dimensional magnonic crystals

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ABSTRACT

In this work we investigate magnonic band gaps, in the terahertz (THz) frequency range, in periodic and quasiperiodic (Fibonacci sequence) magnonic crystals formed by layers of Cobalt (Co) and Permalloy (Py). Our theoretical model is based on a magnetic Heisenberg Hamiltonian in the exchange regime, together with a transfer-matrix treatment within the random-phase approximation (RPA). For periodic arrangements the bulk band structure is analogous to those found in photonic crystals, while for quasiperiodic multilayers it presents additional pass bands similar to those found in doped electronic materials.

The perspective for the development of new technologies based on magnonic frequency band gaps has attracted much attention in the last decade [1–5]. These band gaps, which constitute intrinsic properties of magnonic crystals (MCs), control how magnons (quantized spin waves) move through these crystals, in analogy with photons in photonic crystals (PCs) [6]. It forms the basis of spintronics, whose technological applications range from magnetoelectronic devices [7] to magnonic waveguides [8], to cite just a few.

Magnetic periodic layered structures have been studied for more than a decade, including the discovery of the giant magnetoresistance effect in three-layer systems containing magnetic and non-magnetic layers [9]. However, it is only recently that Kruglyak and Hicken [10] proposed an experiment to investigate the physical properties of magnonic crystals. They have observed that, similarly to the PCs, the spectrum of magnonic crystals is strongly affected by the presence of magnonic band gaps (MBGs), in which magnon propagation is forbidden (for an up to date review see Ref. [11]). A natural extension of the concept of PCs to MCs is to consider a periodic magnetic permeability function instead of a periodic electric permittivity [12], i.e., one looks for the periodicity of the index of refraction $\eta = \sqrt{\mu}$, with $\epsilon = 1$, in MCs, instead of $\eta = \sqrt{\epsilon}$, with $\mu = 1$, as in PCs. This definition is compatible for spin waves in the magnetostatic regime [13]; for magnons in the exchange regime it is necessary to take into account that the exchange terms of the magnetic materials play the same role as the permittivity function in PCs [14]. A direct consequence of this change is the possibility of band gaps appearing at terahertz frequencies with magnitudes that depend upon the thickness of the layers and the geometry of the MCs.

On the other hand, nonperiodic deterministic (quasiperiodic) structures constitute a quite separate research field [15]. These structures do not have translation symmetry, and were mainly considered as suitable theoretical models to describe the conceptual transition from a perfect periodic crystal to some random structure [16,17]. Previously, MBGs have been studied in quasiperiodic magnetic multilayers, with and without uniaxial anisotropy [18], by using a magnetic Heisenberg Hamiltonian in the RPA approximation, and considering the MBGs only in the k_x-direction, k_x being the in-plane wave vector. We extend this work by considering a periodic and quasiperiodic (Fibonacci type) multilayer system (defining a magnonic crystal) composed of ferromagnetic materials, namely Cobalt (Co) and Permalloy (Py) (see Fig. 1(a)), to present the magnon spectra in a path within the three

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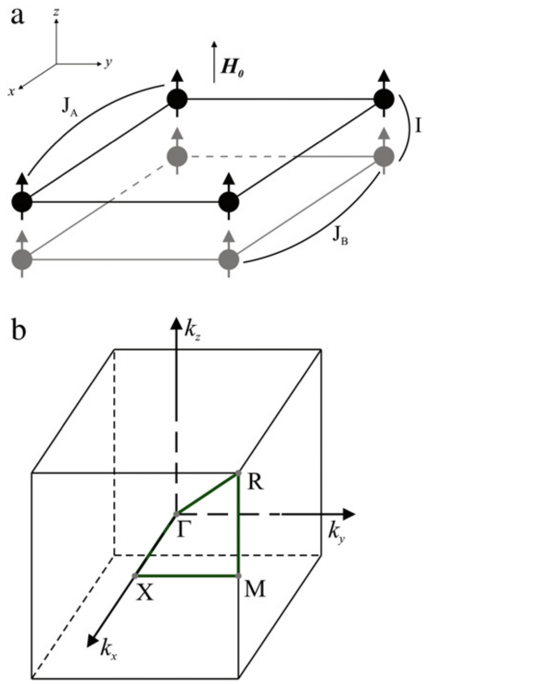

Fig. 1. (a) Schematic representation of the MCs studied here. (b) The reduced Brillouin zone scheme corresponding to the considered structure and the paths (gray lines) along which the magnonic band structures are calculated: $\Gamma \to X$, $X \to M$, $M \to R$, and $R \to \Gamma$.

dimensional reciprocal space, namely, $\Gamma \to X, X \to M, M \to R, R \to \Gamma$ (see Fig. 1(b)). We focus our attention on the THz regime (which has recently become of great interest in connection with a wide range of technological applications, including high bandwidth signal processing, THz imaging, and THz spectroscopy [19,20]), considering a reduced Brillouin zone (BZ) scheme to stress the MBGs in the Fibonacci MCs structures.

Initially, we consider MCs in which n_A layers of material A (Cobalt) alternate with n_B layers of material B (Permalloy) in a periodic way. Both materials are taken to be simple cubic spin-S Heisenberg ferromagnets, having exchange constants J_A and J_B and lattice constant a. The exchange term at the A/B interfaces is I (see Fig. 1(a)). The Heisenberg Hamiltonian for each layer is:

$$H = (-1/2) \sum_{i,j} J_\alpha \vec{S}_i \cdot \vec{S}_j - g\mu_B H_0 \sum_i S_i^z, \quad (1)$$

where the sum in the first term is over sites i and the nearest neighbors j; H_0 is a static external magnetic field pointing in the z-direction, and α is equal to A or B. The unit cell size is $D = na$, with $n = \sum_i n_{Ai} + \sum_j n_{Bj}$, where $\sum_i n_{Ai}$ is the number of Cobalt (A) layers and $\sum_j n_{Bj}$ is the number of Permalloy (B) layers in each cell, respectively. The lth unit cell is defined to run from $(l-1)na + a$ to lna. We are interested in the low temperature regime $T \ll T_C$, in which the spins are fully ordered, i.e., $\langle S_z \rangle = S$.

The dispersion equation for a bulk spin-wave in a ferromagnetic medium (A or B) is found, within the random-phase approximation (RPA), from the equation of motion for the operator $S_i^+ = S_i^x + iS_i^y$, i.e.:

$$i\frac{\partial}{\partial t} S_i^+ = g\mu_B H_0 S_i^+ + \langle S^z \rangle \sum_{n.n.} J_\alpha (S_i^+ - S_j^+), \quad (2)$$

where we have used $\hbar = 1$, for simplicity.

The spin-wave dispersion relation in a periodic MC can be found by solving the RPA equations of motion for the operators S_i^+, using appropriate boundary conditions at the interfaces. A spin that is not at any interface has the same nearest-neighbor environment, and so it has the same equation of motion as a spin in the corresponding bulk medium. Thus, the spin-wave amplitudes are given, within each material, by a linear combination of the positive-and negative-going solutions for the bulk medium, i.e., for a spin localized in the j site, we have:

$$S_j^+ = \{A_l^\alpha \exp[i\vec{k}_i \cdot (\vec{r} - \vec{r}_{l\alpha})] + B_l^\alpha \exp[-i\vec{k}_i \cdot (\vec{r} - \vec{r}_{l\alpha})]\} \exp(-i\omega t), \quad (3)$$

where A_l^α and B_l^α are the amplitudes of the spin-waves for the lth unit cell in material α. Here $\vec{r}_{lA}$ and $\vec{r}_{lB}$ are the positions of the left-hand layers of the corresponding component in cell l, i.e., $\vec{r}_{lA} = [(l-1)\, na + a]\,\hat{z}$ and $\vec{r}_{lB} = [(l-1)\, na + (n_1 + 1)\, a]\,\hat{z}$. Note that there exist a translational symmetry in the xy-plane, and therefore for a traveling wave in the MCs, $k_x = k_{Ax} = k_{Bx}$ and $k_y = k_{Ay} = k_{By}$ must be real. However, k_{Az} and k_{Bz} are only real when ω lies within a pass band for the corresponding bulk mode. Outside the pass band, k_{Az} and k_{Bz} are either imaginary or complex of the form $\pi/a + i\beta$, where β is real.

In the periodic lattice of Fig. 1(a), the equation of motion (2) at the interface A/B relates the amplitudes A_l^B, B_l^B to A_l^A, B_l^A, while when applied at the interface B/A it relates the amplitudes A_{l+1}^A, B_{l+1}^A to A_l^B, B_l^B. We can easily relate the amplitudes (A_l^A, B_l^A) to (A_{l+1}^A, B_{l+1}^A) by the following matrix equations:

$$\begin{pmatrix} A_{l+1}^A \\ B_{l+1}^A \end{pmatrix} = N_A^{-1} M_B N_B^{-1} M_A \begin{pmatrix} A_l^A \\ B_l^A \end{pmatrix}. \quad (4)$$

By using Bloch's theorem, we can show that

$$\cos(QD) = (1/2)\mathrm{Tr}\,[T]\,, \quad (5)$$

with $T = N_A^{-1} M_B N_B^{-1} M_A$. The form of the matrices M's and N's can be found elsewhere [18]. Of course T is a transfer-matrix since it relates the coefficients of the $(l+1)$th unit cell to the coefficients of the lth unit cell. The last equation is written using the fact that T is a unimodular 2×2 matrix. Eq. (5) describes the bulk modes of a spin-wave in a periodic arrangement of the magnetic layers. Once we know the form of the transfer-matrix T, the bulk spin wave spectra are determined.

We now intend to investigate the bulk spin waves in Fibonacci one-dimensional magnonic quasicrystals, by using the calculations described above. We consider that the MQCs are defined like the MCs, the main difference being the spatial distribution of the exchange terms, which are here organized in a quasiperiodic fashion obeying a Fibonacci rule. Based on previous calculation, one can show that the transfer-matrix for any Fibonacci generation can be obtained by [21]:

$$T_{S_m} = T_{S_{m-2}} T_{S_{m-1}}, \quad m \geq 3, \quad (6)$$

where S_m is the mth Fibonacci generation sequence. The initial conditions are $T_{S_1} = N_{AA}^{-1} M_{AA}$ and $T_{S_2} = N_A^{-1} M_B N_B^{-1} M_A$. From the knowledge of these transfer matrices, we can now calculate the spin wave spectra (bulk modes) for these artificial structures by using Eq. (5).

Now we present some numerical calculations to illustrate the magnonic band structure for MQCs as a plot of the frequency ω (in THz) versus the dimensionless in-plane wave vector $k_x a$. The exchange term J in each material are determined by using the expression of the exchange constant per length unit $A = NJS^2/a$ [22], where N is the number of atoms at the edges of a microscopic unit cell in a given layer multiplied by the number of layers in the superlattice unit cell. The physical parameters used are $n_A = n_B = 4$, $S_A = 5/2$, $S_B = 3/2$, and the lattice constant $a = 2.5$ nm. Furthermore, $A_A = 2.88 \times 10^{-11}$ J/m for Co, and $A_B = 1.11 \times 10^{-11}$ J/m for Py [23]. We define the exchange term

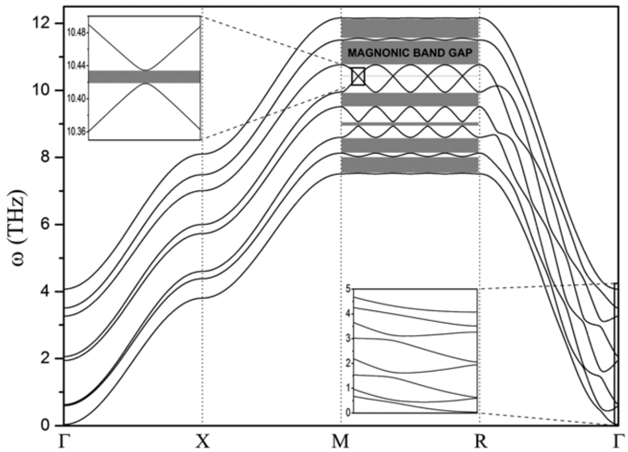

Fig. 2. Band structure of a MC, considering the reduced Brillouin zone, presented in Fig. 1(b). The shaded areas represent the partial magnonic band gaps. Note that the widths of the magnonic band gaps are independent of the wavevector in the $M \to R$ direction (z-direction in real space). The inset on the right-hand side shows the zero slope of the magnon excitation at the edge of the BZ, while the inset on the left-hand side magnifies the narrow band gap, difficult to see in the large scale.

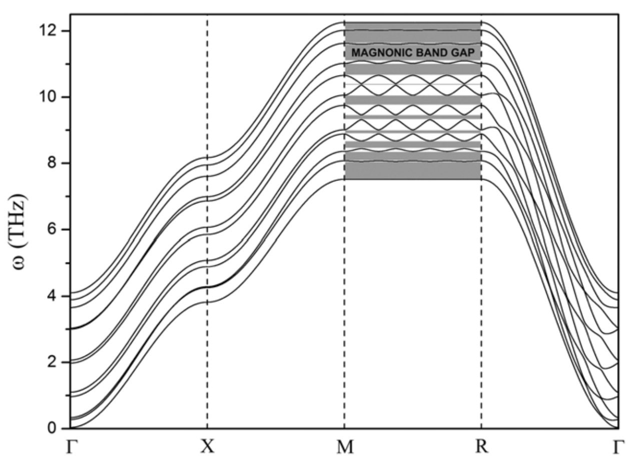

Fig. 3. Same as in Fig. 2, but for the third generation of the Fibonacci MQCs. Here the MBGs are narrower than in the periodic case, and the number of allowed bands is $4 \times F_3 = 12$.

in the interface A/B to be $I = (J_A + J_B)/2$. Finally, the applied static magnetic field is $H_0 = 0.2$ T.

Note that, in the directions k_x and k_y, the limit of the BZ is π/a, where a is the lattice constant. Moreover, in the z-direction, by using Eq. (5), the limit of the BZ is π/D. In order to show all three directions at the same scale, as in the PCs cases, we extend the BZ in the z direction to π/a, without loss of generality.

In Figs. 2 and 3 we present the magnonic band structures in the reduced Brillouin zone for the periodic case as well as to the 3rd generation of the Fibonacci sequence, respectively. First, we observe that the allowed frequencies (and, consequently, the MBGs) are in the THz range. The inset at the right-hand side of Fig. 2 confirms that the slope of the magnon excitation is zero at the edges of the BZ, while the inset at the left-hand side magnifies the narrow band gap, difficult to see in the large scale. In the path $\Gamma \to X$ we have $QD = k_y a = 0$, and a variable $k_x a$ (from 0 to π), whose magnon spectrum is represented by several single lines (the $QD = 0$ part of Fig. 4.6 in Ref. [24]). A similar spectrum is found for the path $X \to M$, where we have $QD = 0$, $k_x a = \pi$, and a variable $k_y a$ (from 0 to π). The magnonic band gaps appear explicitly in the path $M \to R$, where we have $k_x = k_y = \pi/a$, and a variable QD (from 0 to π). They are forbidden regions for the propagation of the magnon bulk modes, and are shown shaded. There are no complete magnonic band gaps (i.e., a frequency region where the gap can extend to cover all possible propagation directions). This result was expected because the change in the translational symmetry occurs only in one direction, which is the z-axis. Between them, there are allowed magnonic bulk bands bounded by the curves $QD = 0$ and $QD = \pi$. Note that, as a consequence of the extension of BZ in the z-direction, we have the allowed magnonic bands represented by a periodic oscillation (repetition of information), which corresponds to the z-direction in real space. The maxima and minima of this periodic oscillation are equivalent to the solutions $QD = 0$ and $QD = \pi$ in Eq. (5), and the amplitudes correspond to the band width in the representation $\omega \times k_x a$ [21]. Finally, in the path $R \to \Gamma$ we have all three dimensionless wavevectors, namely QD, $k_x a$, and $k_y a$, varying from π to 0. There are band gaps, but they are difficult to see because the magnon spectrum in the three-dimensional space (ω, $k_\parallel a$, QD) is now projected into its mid-plane (ω versus $Q = k_\parallel$) and not on the usual (ω versus QD) or (ω versus $k_x a$) planes, where $\vec{k}_\parallel = (k_x, k_y)$.

The spectrum in Fig. 3 displays narrower band gaps in comparison with Fig. 2, due to the growth of the unit cell. The number of modes is related to the number of layers in each material, and it increases with $4 \times F_m$, F_m being the Fibonacci number, while the number of band gaps is $(4-1) \times F_m$ (m is the Fibonacci generation number). Looking at the lower magnonic band gap region in Fig. 3, we can observe the appearance of additional pass bands similar to those found in doped electronic materials. We have also noted that in our discrete model the number of partial band gaps is related to the number of layers in each material, not found in the continuous model, whose main feature is the presence of many band gaps [25,26].

In summary, we presented here a new way to investigate magnon propagation in MCs and MQCs in the exchange regime. For the MCs, we compare the typical magnonic band structure with the PCs one, showing that the MBGs will appear only in the $M \to R$ directions. For MQCs, we have found narrow magnonic band gaps, whose number of modes is related not only to the number of layers in each material (n_A and n_B), but also to the Fibonacci number F_m.

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