Device-independent test of a delayed choice experiment

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The wave or particle duality has long been considered a fundamental signature of the nonclassical behavior of quantum phenomena, especially in a delayed choice experiment, where the experimental setup revealing either the particle or the wave nature of the system is decided after the system has entered the apparatus. However, as counterintuitive as it might seem, usual delayed choice experiments do have a simple causal explanation. Here, we take a different route and under a natural assumption about the dimensionality of the system under examination, we present an experimental proof of the nonclassicality of a delayed choice experiment based on the violation of a dimension witness inequality. Our conclusion is reached in a device-independent and detection loophole-free manner, that is, based solely on the observed data and without the need of special assumptions about the measurement apparatus.

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I. INTRODUCTION

The wave or particle nature of light is amongst the oldest debates in physics [1]. The famous double-slit experiment demonstrating interference [2] settled the question for a while. However, with the establishment of quantum mechanics, the conundrum was back again, giving rise to one of the most counterintuitive features of quantum theory, the wave-particle duality [3,4]; i.e., depending on the experimental apparatus, a quantum system can exhibit a particle behavior or a wave behavior. For instance, a photon in a Mach-Zehnder interferometer displays interference (wavelike) or no interference (particlelike) depending on whether a second beam splitter is put at the intersection of the two interferometric arms. Wave and particle are complementary concepts, and, as argued by Bohr [3], one cannot assign to each quantum system a definite and immutable label that determines its character.

In order to rule out an interpretation in which a quantum system is intrinsically *either* a wave *or* a particle, Wheeler proposed his famous *delayed choice experiment* [3,5], which requires the experimental arrangement, revealing or not the interference pattern, to be decided *after* the photon has entered the interferometer. Over the years, the delayed choice experiment idea has been explored in several directions [6,7], including implementations on different physical platforms [8–10] and generalized to a quantum delayed choice experiment [11–14], where the presence or absence of the second beam splitter is decided by a photon in a quantum superposition. Following that, wave-particle duality has also been related to quantum entanglement [15–18]. Excluding retrocausality, Wheeler and the quantum delayed choice experiment were devised to show that any model whereby the photon has a definite intrinsic wave or particle nature is incompatible with the quantum predictions. Nonetheless, recent results have shown that a delayed choice experiment can be seen as a particular instance of a *prepare and measure* scenario [19,20], and investigations based on causal modeling have revealed that a classical hidden variable (HV) model can in fact yield the results of Wheeler's delayed choice experiment [21,22]. However, it has also been shown that small modifications of the original proposal can rule out significant classes of HV models [22]. The causal modeling approach [23] offers a way to circumvent the debatable notion of wave-particle duality and probe the nonclassical nature of an experiment based solely on the empirical data at hand and some mild causal assumptions. Not requiring assumptions on the employed devices present in the causal structure, apart from the dimension of the HV, it is commonly referred to as device-independent (DI) and provides tools with a vast range of applications [24–32].

In this framework, apart from the assumption of nonretrocausality, the HV is also assumed to have the same dimension as the quantum system it is supposed to mimic. Thus, the nonclassicality in the delayed choice experiment can be tested via well-developed dimensional witness inequalities [19,20] that, if violated, prove that any classical explanation requires a physical system with a dimension higher than that of its quantum counterpart, or else one has to resort to retrocausal influences.

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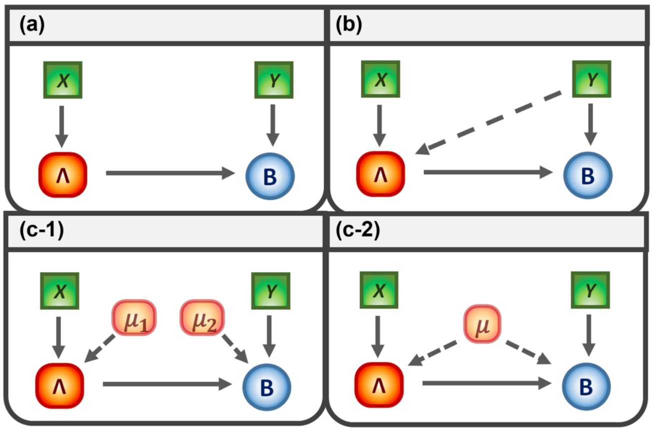

FIG. 1. Causal structures representing the delayed choice experiment (a) *Prepare and Measure* scenario, where a classical variable Λ or a quantum state ρ depending on whether we refer to classical or quantum description, depends on a classical variable X, while the choice of which measurement to perform is denoted by Y. Under the assumption of nonretrocausality, the variables Y and Λ should be statistically independent; that is, $p(\lambda, y) = p(\lambda)p(y)$. (b) Relaxation on the assumption of no retrocausality in a prepare and measure scenario: to explain the violation of the dimensional witnesses reported through a classical HV model, we need to allow retrocausality. (c) Prepare and measure scenario affected by noise terms. (c-1) The preparation and measurement devices are uncorrelated, since they are affected by two independent noise terms, μ_1 and μ_2. (c-2) Preparation and measurement stages are allowed to have pre-established correlations mediated by the variable μ.

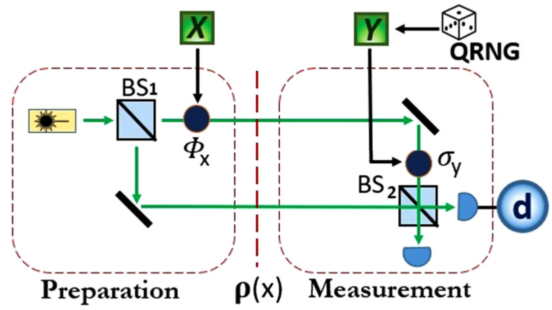

FIG. 2. Delayed choice experiment as a prepare and measure scenario through a Mach-Zehnder interferometer. Dotted squares on the left and on the right inglobe respectively the preparation and measurement stages. X and Y select respectively among three and two phases, which are added on one arm of the interferometer. The output b is 1 when the highlighted detector clicks and 0 when it does not.

Here, we report on the realization of device-independent tests of nonclassicality in two complementary photonic quantum delayed choice experiments. First, under the reasonable assumption that the preparation and measurement stages in our experiment are uncorrelated [see Fig. 1(c1)], we provide an experimental detection loophole-free violation of a dimensional witness. Second, we provide more resources for the potential HV model, relaxing the hypothesis of independent noise terms affecting the preparation and measurement devices [see Fig. 1(c2)]. Even in this case, under the fair-sampling assumption, we can violate a dimensional witness inequality. Altogether, our results provide a DI verification of the true quantum nature of delayed choice experiments, without the need to resort to quantum entanglement [15–17]. Indeed, only under the assumption of no retrocausality (and independence of measurement and preparation devices, depending on which inequality we are testing), the proposed tests unambiguously prove that any classical description of the experimental data necessarily needs a system with more than two dimensions.

II. A DELAYED CHOICE EXPERIMENT AS A PREPARE AND MEASURE SCENARIO

As recently noted in Ref. [22], a delayed choice experiment can be seen as a particular instance of a prepare and measure scenario. Upon receiving an input x, a state preparation device emits a quantum state ρ_x (Fig. 2). This state is then sent to a measurement device, where the measurement to be performed is selected by another input y, producing output b. The quantum description of the experiment produces probability distributions according to the Born rule $P_Q(b|x, y) = \mathrm{Tr}(\rho_x M_y^b)$, where M_y^b is a positive operator such that $\sum_b M_y^b = \mathbb{1}$, for all y. In turn, in a classical description modeled by a HV causal model, the probability can be decomposed as $P_C(b|x, y) = \sum_\lambda p(\lambda|x)p(b|\lambda, y)$, where the classical variable λ represents the value of the hidden variable Λ describing the behavior of the photon in the interferometer. The central causal assumption in a prepare and measure scenario as well as in a delayed choice experiment is the fact that the choice Y does not have causal influence over the preparation stage of the state ρ_x (or Λ in the HV description, see Fig. 1). If there are no bounds on the Hilbert space dimension or the cardinality of λ is limited, there are quantum distributions that cannot be generated and require a classical model with higher dimensionality [19,20,22].

The application of this approach to Wheeler's delayed choice experiment and the quantum delayed choice experiment—where the measurement choices correspond to the presence or absence of a second beam splitter—shows that the HV model with dimension equal to $d = 2$ can reproduce the quantum predictions [22]. Indeed, as expected, since the delayed choice experiment involves a single photon in two modes, encoding at most a single classical bit [33], Λ should likewise be binary. However, if instead the second beam splitter is always present and the measurement choice now corresponds to an extra phase σ_y (imprinted long after the photon has entered the interferometer), two-dimensional HV models can no longer reproduce the quantum correlations.

Our setup, depicted in Fig. 3, is a variation of the apparatus shown in Fig. 2, involving the equivalent in polarization of a Mach-Zehnder interferometer, composed of two half-wave plates (HWP) [see Fig. 3(a)]. The first acts as the initial beam splitter of a Mach-Zehnder interferometer, generating a superposition of the two orthogonal polarizations mimicking the spatial paths, while the second takes the place of the final beam splitter, recombining the polarizations and allowing interference between them.

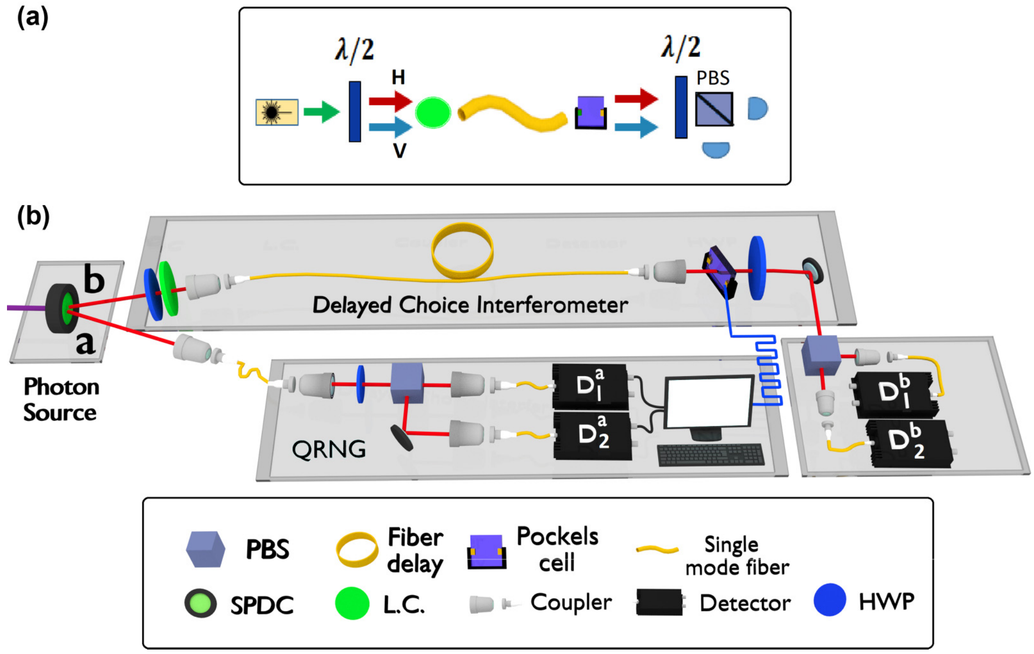

FIG. 3. Experimental apparatus for the violation of the dimensional witness. (a) A pictorial representation of our apparatus, exploiting the equivalent, in polarization, of a Mach-Zehnder interferometer. (b) One of the generated photons (trigger) is sent to the quantum random number generation (QRNG) stage (path a) to select the phase shift σ_y, between two values σ_1 and σ_2. The action of a rotated HWP before the PBS ensures an intrinsically random and uniformly distributed choice. When D_1^a clicks, it triggers the application of a voltage on the Pockels cell on path b, which adds the required phase shift σ_1, otherwise $\sigma_y = \sigma_2 = 0$. The second photon (vertically polarized) enters the polarization Mach-Zehnder interferometer. A 22.5°-rotated HWP puts the photons in a superposition of polarization components; a liquid crystal (LC) introduces a phase shift ϕ_x between the polarization components. The photon is delayed 710 ns, employing a 140-m-long single-mode fiber, and passes through the Pockels cell, which, when triggered, introduces the $\pi/2$ phase shift; finally another 22.5°-rotated HWP closes the interferometer.

In the preparation stage, after the first HWP, the variable X randomly selects a phase ϕ_x inserted between the two polarizations via a liquid crystal (LC). In the measurement stage, to ensure the *delayed choice*, we employ two key elements: (i) a quantum random number generator (QRNG) based on a measurement performed on a single photon, which randomly impinges on the extra phase σ_y, and (ii) a fast electro-optics modulator (Pockels cell) implementing this phase within the Mach-Zehnder interferometer. The measurement is realized by a polarizing beam splitter (PBS) whose outputs are sent to two detectors. With this setup we have violated two distinct types of dimension witness inequalities. The measurements in both inequalities are performed by an extra phase corresponding to $\sigma_1 = \pi/2$ or $\sigma_2 = 0$.

The QRNG is implemented through a horizontally polarized photon (trigger) sent on path a, where it is rotated by 45° and consecutively measured in order to randomly select between σ_1 or σ_2 after the signal photon enters the Mach-Zehnder interferometer (see Appendix A). The trigger (a) and signal (b) photons are produced by the same source (Appendix B), opening a possible loophole: one could imagine a scenario where a hidden variable on photon a is a common cause of the probability of triggering detector D_1^a and the probability of triggering detector D_1^b. Then we have to assume that the signal and trigger polarizations are independent (this is reasonable since they are not entangled) in order to consider the choice of measurement Y independent from the signal behavior. The Pockels cell allows the choice between σ_1 and σ_2 to be made swiftly, since its response time is within the order of nanoseconds (see Appendix C). When the detector D_1^a clicks, the signal is split, so it can both be registered by the coincidence counter and trigger the Pockels cell that introduces a $\pi/2$ phase shift between the two superimposed polarizations. When D_2^a clicks, no voltage is applied and so no extra phase is introduced.

First, we have tested the W_2 dimensional witness allowing us to rule out in a DI manner any two-dimensional HV model where the preparation and measurement devices are independent [see Fig. 1(c1)]. Its experimental value is obtained by selecting four possible preparations x and two possible measurements y, as the determinant of the following 2×2 matrix:

$$W_2 = \begin{pmatrix} p(1,1) - p(2,1) & p(3,1) - p(4,1) \\ p(1,2) - p(2,2) & p(3,2) - p(4,2) \end{pmatrix}, \quad (1)$$

where $p(i,j) = p(b = 1|x = i, y = j)$, with $i = 1, \dots, 4$ and $j = 1$ or 2. Any two-dimensional classical system satisfies $|\det(W_2)| = 0$. In our setup, selecting the following values for ϕ_x and σ_y: $\phi_1 = 0$, $\phi_2 = \pi$, $\phi_3 = -\pi/2$, and $\phi_4 = \pi/2$, and $\sigma_1 = \pi/2$ and $\sigma_2 = 0$, the quantum probabilities achieve $|\det(W_2)| = 1$, thus violating the classical prediction. Experimentally, we obtained $|\det(W_2)| = 0.951 \pm 0.010$, postselecting only those events for which there is a simultaneous arrival of a signal photon and a trigger photon. This is equivalent to having detectors with the highest efficiency possible, $\eta = 1$, and ideal transmittances, $T_a, T_b = 1$. Under this fair-sampling assumption, we say that $b = 1$ when we have a coincidence between the trigger photon and D_1^b, and we say that $b = 2$ when the coincidence is with D_2^b. However, W_2 can also be violated in a detection loophole-free manner. To this aim, we consider each click of the trigger photon as an experiment run and assign to b the value of 2 when a coincidence is registered with D_2^b and a value of 1 both when the coincidence is with D_1^b and when the photon is lost.

In this case, the real efficiency of D_1^b and eventual imperfections in the transmittances affect the witness value, which is now given by $|\det(W_2^{\eta,T_a,T_b})| = \eta^2 T_a^2 T_b^2 |\det(W_2^{\eta=1,T_a,T_b=1})|$ and thus in principle can be violated for any $\eta, T_a, T_b > 0$. Indeed, we obtained a violation given by $W_2 = (1.23 \pm 0.03)10^{-4}$ and demonstrated that the obtained statistics cannot be explained by a classical two-dimensional HV model even in the presence of major experimental imperfections. We have also tested stronger HV models where the preparation and measurement devices can be correlated [see Fig. 1(c-2)]. Note that such allowed correlations are restricted to those shown in Fig. 1(c-2) and the choices of measurement Y and preparation X are free and independent of the variable μ mediating any potential correlation between the devices. Since the delayed choice experiment is a timelike scenario, differently from a Bell scenario, the independence of X and Y cannot be enforced by a spacelike separation. Notice that variants of a delayed choice experiment where one indeed can impose spacelike separation between X and Y have been proposed [18], however, requiring quantum entanglement and a very different causal structure from the one analyzed here. Within a scenario with three possible x inputs, while y is still dichotomic we consider the following dimension witness [19]:

$$I_{\rm DW} = |\langle B_{11}\rangle + \langle B_{12}\rangle + \langle B_{21}\rangle - \langle B_{22}\rangle - \langle B_{31}\rangle|, \quad (2)$$

where $\langle B_{xy}\rangle = p(b = 1|x, y) - p(b = 2|x, y)$, with $x = 1, 2,$ and 3, and $y = 1$ and 2. The classical bound for $I_{\rm DW}$ is 3, while the quantum bound is $Q_2 = 1 + 2\sqrt{2} = 3.828$ [19,22,34]. A value of $I_{\rm DW}$ higher than the classical bound thus rules out any two-dimensional HV model that does not allow for retrocausality, even with no special assumptions on the potential pre-established correlations. The choices of ϕ_x and σ_y which maximize $I_{\rm DW}$ are the following: $\phi_1 = 7/4\pi$, $\phi_2 = 5/4\pi$, and $\phi_3 = \pi/2$, and $\sigma_1 = \pi/2$ and $\sigma_2 = 0$.

To experimentally find violations of the witness, we measured $I_{\rm DW}$, with different preparation settings. In particular we selected 70 different values for each one of the three preparation phases, distributed in an approximately uniform way in the interval $[0, 2\pi]$. Then we evaluated all the possible 70^3 values of $I_{\rm DW}$, each corresponding to a different three-tuple of ϕ_x, always with $\sigma_1 = \pi/2$ and $\sigma_2 = 0$. As shown in Fig. 4(b), for some configurations, we are able to violate the classical bound and obtain values within the range between 3 and $1 + 2\sqrt{2}$. Specifically, in blue, we plot the experimental probability distribution to obtain each $I_{\rm DW}$ value and highlight the classical (pink) and quantum (purple) upper bounds. This procedure, where we span different values of ϕ_x, highlights the device-independent nature of our test, since it does not require the trust in the experimental apparatus to insert exactly a ϕ_x phase shift between the two interferometric arms.

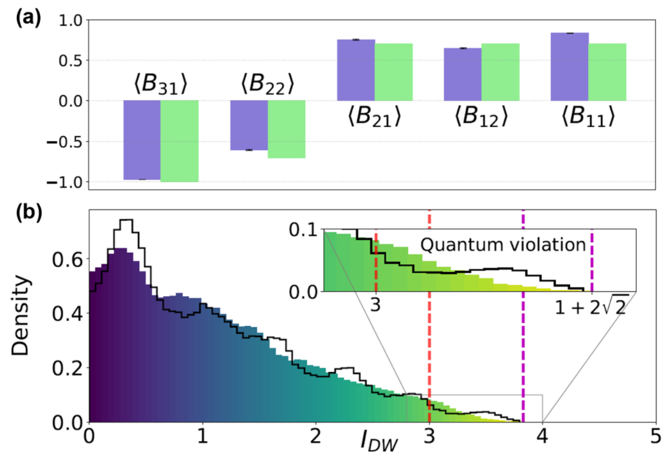

FIG. 4. Experimental violation of dimensional witness $I_{\rm DW}$. (a) Experimental values (purple bars, to the left) of each of the terms in Eq. (2). Error bars indicate 1 s.d. of uncertainty, due to Poissonian statistics. The corresponding theoretical values (green bars, to the right) for the optimal tuple of ϕ_x $(7/4\pi, 5/4\pi, \pi/2)$ are respectively $(1/\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2}, -1/\sqrt{2},$ and $-1)$. (b) Histogram of the experimental probability distribution to obtain each value of $I_{\rm DW}$, from 0 (lowest value) to 5 (postquantum bound); each bin has a width of 0.055. The dashed pink line indicates the classical limit of $I_{\rm DW}$, 3, while the purple one indicates the quantum limit, $1 + 2\sqrt{2} \approx 3.828$. The bars' colors vary along with the values of $I_{\rm DW}$ on the x axis, which correspond to the 70^3 roughly uniformly chosen tuples of phase shifts ϕ_x. Some of these tuples present a violation of the classical limit, as can be seen in the enlarged region of the histogram. The black line is the theoretical prediction of the distribution in which we take into account the nonperfect action of the Pockels cell when working in $\sigma_1 = \pi/2$.

The highest experimental $I_{\rm DW}$ value is 3.822 ± 0.011, as reported in Fig. 4(a). In order to explain this result within a two-dimensional classical framework, we would need to relax the assumption of complete absence of influence between Y and Λ, that is, to allow retrocausality. As detailed in Ref. [22], the minimum amount of retrocausality as quantified by the measure $R_{Y\to\Lambda}$ is given by $\min(R_{Y\to\Lambda}) = \max[\frac{I_{\rm DW}-3}{4}, 0]$. Thus, our experiment indeed falsifies even stronger models where retrocausality is allowed but below the threshold $R_{Y\to\Lambda} = 0.2055 \pm 0.0027$.

III. DISCUSSION

In this work we realized a modified delayed choice experiment [22] that allows one to discriminate, in a device independent way and without the need of entanglement, quantum predictions from those of a nonretrocausal HV theory. Given the causal structure of the prepare and measure scenario, the only crucial assumption is that the dimension of the HV is dichotomic (e.g., assuming "wave" or "particle" values); i.e., it has the same dimension as the quantum system under examination. Our experiment is based on the equivalent, in polarization, of a Mach-Zehnder interferometer. In turn, the delayed choice of the observables is achieved by a quantum random generator and an active feed-forward which triggers a fast electro-optical modulator. With this setup, we measured two DI dimensional witnesses: (i) W_2, whose experimental value overcomes the classical expected value of 0 by 41 standard deviations (s.d.), and (ii) I_{DW} whose experimental value violates the inequality by more than 70 s.d., being compatible with the quantum predictions.

The violation of W_2 is achieved in a detection loophole-free manner; i.e., there is no need to impose the fair-sampling assumption, as the inequality can be violated for any losses and inefficiencies, but requires the assumption of independent preparation and measurement devices, an assumption that can be relaxed in the violation of I_{DW} (requiring the fair-sampling assumption). In this framework the demonstration does not depend on the particular definition of wave-particle objectivity and then we do not need to insert or remove the final beam splitter in order to demonstrate the incompatibility of the photon behavior with a classical explanation. It is more general since it is able to rule out any two-dimensional HV (regardless of its labels: wave or particle, 0 or 1, and so on) accounting for the statistics of single photons in a Mach-Zehnder interferometer.

It is also important to note that, unlike the Bell scenario that is a spatial scenario, this delayed choice experiment is mapped in a prepare and measure scenario that is a temporal scenario with different assumptions with respect to Leggett's crypto-nonlocality [35] or others based on quantum contextuality [36,37]. In other terms, in a Bell scenario to fulfill the locality assumption, necessarily the two measurement devices need to be spacelike separated, while in a prepare and measure scenario we have by definition a timelike separation between the preparation and measurement devices. Contrary to previous experiments that have violated dimension witnesses in a prepare and measure scenario (e.g., Refs. [38–40]), our experiment applies, for the first time, these tests in a delayed choice configuration in which the choice of measurement is made after the preparation. Finally, our results highlight the relevance of revisiting foundational experiments from a causal perspective and we expect it might trigger further applications of causal modeling to other fundamental tests as well as to applications, such as randomness generation [31] and certification [41].

Note added. Recently, we became aware of a similar modified delayed choice experiment of H.-L. Huang *et al.* [42].

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APPENDIX A: QUANTUM RANDOM NUMBER GENERATION

The QRNG is implemented through a horizontally polarized photon (trigger), sent on path a. Its polarization is rotated by a HWP to generate $|+\rangle = (|H\rangle + |V\rangle)/2$, and it is then sent through a PBS. When the emerging photon is horizontally (vertically) polarized, the selected phase shift is σ_1 (σ_2). This procedure gives the same probability to both σ_1 and σ_2, ensuring an intrinsically random choice. The trigger photon is generated at the same time as the photon which enters the Mach-Zehnder interferometer (signal) and thus can guarantee that the measurement choice is performed after the signal photon has entered the interferometer.

APPENDIX B: EXPERIMENTAL DETAILS

Photon pairs were generated in a periodically poled potassium titanium phosphate (ppKTP) nonlinear crystal injected by a continuous pump field with $\lambda = 404$ nm. After spectral filtering, photons were sent to the "delayed choice interferometer" station and to the QRNG station depicted in Fig. 3. The probability of four photon or higher order emissions is negligible in the pump power range of our experiment, that is of 12 mW in continuous mode. The ultimate rate, after the many couplings, was near 5 kHz of singles on detector D_1^b and near 125 Hz of coincidences.

APPENDIX C: FEED-FORWARD OF INFORMATION

The crystal used to implement active feed-forward is a LiNbO3 high-voltage micro Pockels cell made by Shangai Institute of Ceramics with <1 ns rise time, and a fast electronic circuit transforming each Si-avalanche photodetection signal into a calibrated fast pulse in the kilovolt range needed to activate the Pockels cell is fully described in Ref. [43]. To achieve the active feed-forward of information, the photon sent to the delayed choice interferometer station in Fig. 3(b), needs to be delayed, thus allowing the measurement on the first photon to be performed. The amount of delay was evaluated considering the velocity of the signal transmission through a single-mode fiber and the activation time of the Pockels cell. We have used a fiber 140 m long, coupled at the end into a single-mode fiber that allows a delay of 720 ns of the second photon with respect to the first.

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