Optical transmission spectra in symmetrical Fibonacci photonic multilayers

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Abstract:
We study the transmission properties of light through the symmetric Fibonacci photonic multilayers, i.e., a binary one-dimensional quasiperiodic structure, made up of both positive (SiO2) and negative refractive index materials with a mirror symmetry. These spectra are calculated by using a theoretical model based on the transfer matrix approach for normal incidence geometry, in which many perfect transmission peaks (the transmission coefficients are equal to the unity) are numerically obtained. Besides, the transmission coefficient exhibits a six-cycle self-similar behavior with respect to the generation number of the Fibonacci sequence.

1. Introduction

The intriguing physics of a medium having a negative refractive index was theoretically discussed by Veselago almost four decades ago, by means of a hypothetical material possessing simultaneously negative magnetic permeability \( \mu \) and electric permittivity \( \epsilon \) [1]. He coined these peculiar materials as Left-Handed Material (LHM) because they support backward waves, i.e., \( \vec{E} \), \( \vec{H} \) and \( \vec{S} \) form a left-handed triplet.

This rather academic investigation became a seminal research letter when Pendry and collaborators [2–4] demonstrated theoretically that, in the microwave region, a lattice of metallic split-ring resonators (SRRs), with characteristic features in the millimeter range, behaves as an effective medium with a negative magnetic permeability \( \mu_{\text{eff}} \). Furthermore, a network of thin metallic wires behaves as a quasi-metal with negative permittivity \( \epsilon_{\text{eff}} \) at microwave frequencies [3–5]. By combining these two structures, Smith et al. [6] fabricated a material which, within a certain frequency range, has both \( \mu \) and \( \epsilon \) negative, a true LHM media.

Although some of the properties of LHM are yet not fully understood, they offer a rich ground for both theoretical and experimental research [7–9]. More recently, search for such materials in the optical region has attracted a great deal of attention, and has been proposed that photonic gap materials can behave as an effective LHM at optical frequencies [10]. Analogous to Bloch electron waves in the band structure of a crystal, optical waves in the periodic lattice of a photonic gap material can have Bloch states with its wave vector and group velocity in opposite directions [11–13].

The so-called photonic crystals are periodic dielectric or metallic structures, whose photonic bands exhibit arbitrarily different dispersions for the propagation of electromagnetic waves, with forbidden band gaps at certain range of wavelengths. In this respect, there is a close analogy between a photon in a photonic crystal, and an electron in a semiconductor material. Based on these properties, photonic crystals is a medium where the propagation of light can be modified virtually in any way, although in a controllable manner, providing access to novel and unusual optical phenomena [14–16].

It is the aim of this work to investigate the transmission spectra of a light beam normally incident from a transparent medium into a photonic structure composed of SiO2/LHM multilayers arranged...
in a quasiperiodical symmetrical Fibonacci fashion. Quasiperiodic structures, which can be idealized as the experimental realization of a one-dimensional quasicrystal, are composed from the superposition of two (or more) building blocks that are arranged in a desired manner. The symmetrical Fibonacci multilayer photonic structure can be viewed as a superposition of two Fibonacci superlattices with mirror reflection at the center (see Fig. 1). The structure can be viewed as a superposition of two Fibonacci superlattices with mirror reflection at the center (see Fig. 1). The symmetrical Fibonacci superlattices with mirror reflection at the center (see Fig. 1). The symmetrical Fibonacci superlattices with mirror reflection at the center (see Fig. 1). The symmetrical Fibonacci superlattices with mirror reflection at the center (see Fig. 1).

The isotropic electromagnetic medium can be generally described for the electromagnetic fields. To this end, we consider that a p-polarized (transverse electric wave) light of frequency \( \omega \) is normally incident from a transparent medium \( C \) with respect to the one-dimensional photonic crystal formed by the layered system (see Fig. 1). We choose the z-axis to be normal to the interface, the x-axis to be in the plane of the figure, and the y-axis to be out of the plane of the figure. Here we have considered the p-polarization mode without loss of generality, since, at normal incidence, both s- and p-polarizations give the same results. The isotropic electromagnetic medium can be generally described by a magnetic permeability \( \mu(\omega) \) and dielectric permittivity \( \epsilon(\omega) \). Its dispersion relation can be obtained by solving the wave equation \([21,22]\)

\[
\frac{Z(\omega)}{n(\omega)} \frac{d}{d\omega} \left[ \frac{1}{Z(\omega) n(\omega)} \frac{d E(\omega)}{d\omega} \right] = \left( \frac{\omega c}{\beta} \right)^2 E(\omega),
\]

where \( n(\omega) = \sqrt{\epsilon(\omega)} \sqrt{\mu(\omega)} \) and \( Z(\omega) = \sqrt{\mu(\omega)} / \sqrt{\epsilon(\omega)} \) are the refractive index and the impedance at frequency \( \omega \), respectively, which are layer dependent. The medium \( A \), with thickness \( d_A \), is fulfilled by SiO\(_2\) and is characterized by a constant positive refractive index \( n_A \). The medium \( B \), with thickness \( d_B \), is fulfilled by a LHM, characterized by a negative refractive index \( n_B \). They are surrounded by a transparent medium \( C \) with a constant refractive index \( n_C \).

The transmission of a normal incident light wave across the interfaces \( \alpha \rightarrow \beta \) (\( \alpha, \beta \) being any \( A, B \) and \( C \) medium) is defined by the interface matrix

\[
M_{\alpha \beta} = \frac{1}{2} \left( \frac{1 + Z_\alpha/Z_\beta}{1 - Z_\alpha/Z_\beta} \right). 
\]

**2. Transfer matrix approach**

We now intend to investigate the light transmission spectra in artificial structures exhibiting deterministic disorders, i.e., the Fibonacci superlattice with mirror symmetry. To calculate the light transmission rate through the multilayer system, we use a transfer matrix approach for the electromagnetic fields. To this end, we consider that a p-polarized (transverse electric wave) light of frequency \( \omega \) is normally incident from a transparent medium \( C \) with respect to the one-dimensional photonic crystal formed by the layered system (see Fig. 1). We choose the z-axis to be normal to the interface, the x-axis to be in the plane of the figure, and the y-axis to be out of the plane of the figure. Here we have considered the p-polarization mode without loss of generality, since, at normal incidence, both s- and p-polarizations give the same results. The isotropic electromagnetic medium can be generally described by a magnetic permeability \( \mu(\omega) \) and dielectric permittivity \( \epsilon(\omega) \). Its dispersion relation can be obtained by solving the wave equation \([21,22]\)

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\[
M_{\alpha \beta} = \frac{1}{2} \left( \frac{1 + Z_\alpha/Z_\beta}{1 - Z_\alpha/Z_\beta} \right). 
\]
The propagation of the light wave within one of the layers $\gamma$ ($\gamma = A$ or $B$) is characterized by the propagation matrix

$$M_{\gamma} = \begin{pmatrix} \exp(-ik_{\gamma}d_{\gamma}) & 0 \\ 0 & \exp(i k_{\gamma} d_{\gamma}) \end{pmatrix},$$

with $k_{\gamma} = n_{\gamma} \omega / c$.

The transfer matrix for the first Fibonacci sequence, $S_1 = A$, is given by

$$M_1 = M_{CA} T_1 M_{AC}, \quad T_1 = M_A,$$  

where $M_{CA}$ is the propagation through the interface $C/A$, $M_A$ is the propagation in layer $A$, and $M_{AC}$ is the propagation through the interface $A/C$ ($N = 1$ in Fig. 2). The same procedures can be used for the cases with $N = 2$ and $N = 3$, yielding:

$$M_2 = M_{CB} T_2 M_{BC}, \quad T_2 = T_1^2 T_2^R,$$  

$$M_3 = M_{CA} T_3 M_{AC}, \quad T_3 = T_1^3 T_3^R,$$

where

$$T_2^L = M_B M_{BA} M_A, \quad T_2^R = M_A M_{AB} M_B,$$  

$$T_3^L = T_1 M_{AB} T_1^2, \quad T_3^R = T_2 M_{BA} T_1,$$

and the indexes $L$ and $R$ representing the left and right side of the multilayer, respectively (see Fig. 2).

Now it is possible to generalize the expression for the $N$th generation of the transfer matrix for any higher order $N \geq 4$:

$$M_N = \begin{cases} M_{CA} T_N M_{AC}, & \text{for } N \text{ odd,} \\ M_{CB} T_N M_{BC}, & \text{for } N \text{ even.} \end{cases}$$

Here

$$T_N = T_N^L \cdot T_N^R,$$

where

$$T_N^L = \begin{cases} T_{N-2} M_{AB} T_{N-1}^L, & \text{for } N \text{ odd,} \\ T_{N-2} T_{N-1}^L, & \text{for } N \text{ even}, \end{cases}$$  

$$T_N^R = \begin{cases} T_{N-1} M_{BA} T_{N-2}^R, & \text{for } N \text{ odd,} \\ T_{N-1} T_{N-2}^R, & \text{for } N \text{ even}. \end{cases}$$

In order to obtain the transmission spectra, we must relate the amplitudes $A_{1}^N$ and $A_{2}^N$ of the electromagnetic field in the transparent medium $C$ at $z < 0$ with those ones $A_{1}^N$ in the region $z > l$, $l$ being the size of the quasiperiodic structure with mirror symmetry. By successive applications of the above equation in each layer, together with Maxwell’s electromagnetic boundary conditions at each interface along the multilayer system we obtain, for the $N$th generation of the symmetric Fibonacci photonic superlattice,

$$\begin{pmatrix} A_{1}^N \\ A_{2}^N \end{pmatrix} = M_N \begin{pmatrix} A_{1}^N \\ 0 \end{pmatrix},$$

with $M_N = M_{CB} \cdots M_A M_{AB} M_B M_{BA} M_A M_{AB} M_B M_{BA} M_A \cdots M_{BC}$, being the optical transfer matrix of the $N$th-generation quasiperiodic multilayer system, which can be calculated recursively by (9).

The reflectance and the transmittance coefficients are simply given by

$$R = \left| \frac{M_{21}}{M_{11}} \right|^2 \quad \text{and} \quad T = \left| \frac{1}{M_{11}} \right|^2,$$

where $M_{i,j}$ ($i,j = 1,2$) are the elements of the optical transfer matrix $M_N$.

3. Numerical results

In this section we present numerical simulations for the light transmission through the quasiperiodic multilayered photonic structure. We have chosen medium $A$ as silicon dioxide (SiO$_2$), whose refractive index is $n_A = 1.45$, while medium $B$ (LHM) is considered to have $n_B = -1$. Also, we assume the individual layers to be quarter-wave layers, for which the quasiperiodicity is expected to be more effective [23], with the central wavelength $\lambda_0 = 32$ nm ($\approx 58.9$ GHz). These conditions yield the physical thickness $d_j = (8/\pi) \mu m$, $j = A$ or $B$, such that $n_A d_A = n_B d_B$, giving a reversed phase shift in the two materials. Considering medium $C$ to be a vacuum, the phase shifts are given by

$$\delta_A = (\pi / 2) \Omega \cos(\theta_A),$$  

$$\delta_B = (\pi / 2) \Omega \cos(\theta_B),$$

where $\Omega$ is the reduced frequency $\Omega = \omega / \omega_0 = \lambda_0 / \lambda$. For normal incidence, $\theta_A = \theta_B = 0$, and $\delta_A = -\delta_B$. Here, the negative phase shift for medium $B$ means that the light waves propagate in a direction opposite to the energy flux ($+z$-direction in Fig. 1), i.e., one plane light wave, whose electromagnetic field is proportional to $\exp(-i \delta_B)$, propagates in the $-z$-direction, while the Poynting vector propagates in the $+z$-direction. Therefore, inside medium $B$ the effect of the negative refraction index is to change the forward waves $\exp[i(\delta_B)]$ into backward waves $\exp(-i \delta_B)$ and vice versa. This effect keeps the same configuration for the incident and reflected electromagnetic wave at the interface $AB$, but the electromagnetic wave at layer $B$ has now a sign change in the exponentials when compared to the electromagnetic wave at layer $A$.

The optical transmission spectrum for the 16th-generation (3194 layers) of the quasiperiodic Fibonacci sequence with mirror symmetry, as a function of the reduced frequency $\Omega$, is shown in Fig. 3(a). The transmission spectrum presents a unique mirror symmetrical profile around the central peak frequency $\Omega = 1$, which is of course the midgap frequency of a periodic quarter-wavelength multilayer, since in this case the phase-shift $\delta_B = -\pi$. Besides, the structure is transparent (the transmission coefficient is closely equal to 1.0) in the central range of frequency $0.942 < \Omega < 1.058$, and at the reduced frequencies distributed symmetrically at $\Omega = 0, 0.330, 0.487, 0.710, 1.290, 1.513, 1.670$ and 2.0, respectively. The condition of transparence implies that the layers $A$ and $B$ are equivalent from a wave point of view. The photonic band gaps can be better characterized if one consider the narrow frequency range $0.977 \leq \Omega \leq 1.023$ for the optical transmission spectrum, as it is depicted in Fig. 3(b).

The transmission spectrum has scaling property with respect to the generation number of the Fibonacci sequence, within a symmetrical interval around $\Omega = 1$. To understand this scaling property, consider Fig. 4(a), which shows the optical transmission spectrum corresponding to the 22th generation (57714 layers) of the quasiperiodic Fibonacci sequence. Once again, the transmission spectrum presents an unique mirror symmetrical profile around the central peak $\Omega = 1$ with an increase density of peaks producing a dark continuum at the edges of the figure. Narrowing the frequency range, as shown in Fig. 4(b), this spectrum is the same, compared to the one depicted in Fig. 3(b), in the range of frequency amplified by a scale factor equal to 23. It is interesting to notice that this happens every time the difference between two generation numbers of the Fibonacci sequence is six. Note that the self-similarity is not limited to a narrow central region of the spectra, but it embraces several peaks distributed symmetrically around of $\Omega = 1$, giving rise to a much broad spectrum. An important point to consider is that when the generation number $N$ of the Fibonacci sequence increases, the self-similar behavior becomes broader.
Fig. 3. Normal-incidence transmission spectra of a light beam into a Fibonacci multilayered photonic structure with mirror symmetry as a function of the reduced frequency $\Omega = \omega / \omega_0$ for the 16th generation of the Fibonacci sequence: (a) the transmittance $T$ for the range of frequency $0 \leq \Omega \leq 2.0$. (b) same as in (a), but for the range of frequency $0.977 \leq \Omega \leq 1.023$.

Fig. 4. Same as in Fig. 3, but for the 23rd generation of the Fibonacci sequence. The range of frequency in (b) is $0.999 \leq \Omega \leq 1.001$. 
In conclusion, we have studied the light-wave propagation in the symmetric Fibonacci quasiperiodic photonic multilayers. We have shown that the symmetric internal structure in one-dimensional quasiperiodic systems can greatly enhance the transmission intensity, with a striking self-similar behavior occurring every time the difference between two generation numbers of the Fibonacci sequence is equal to six. Our results present some differences from those found for the case of symmetric Fibonacci multilayers with both positive refractive index materials [17–19], for instance, the transmission peaks are different in the form and in the intensity, mainly for low values of the number of generation of the Fibonacci’s sequence.

The above discussions apply only to the ideal case where both the magnetic permeability $\mu$ and the electrical permittivity $\epsilon$ are frequency non-dispersive, which is valid under the assumption that the size of the fabricated negative refractive index material can be as tiny as the normal positive refractive index material. Recently, we have studied the optical transmission in non-symmetrical multilayered Fibonacci system with artificial negative refractive index left-handed material that have the magnetic permeability $\mu$ and the electrical permittivity $\epsilon$ frequency dispersive, being simultaneously negative only within a narrow frequency bandwidth (typical frequency region ranges from 1 to 14 GHz) [24,25]. For this more realistic model we have a rich transmission profile of Bragg peaks with no more similarity or mirror symmetry in their optical transmission spectra.

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